Effect of boundary conditions on the dynamics of a pulse solution for reaction-diffusion systems

Shin-Ichiro Ei
Department of Mathematical Sciences,
Faculty of Mathematics, Kyushu University
Fukuoka 812-8581, Japan

Reaction diffusion systems have been widely treated to describe and study spatio-tempral patterns in dissipative systems. Among them, many reaction-diffusion systems which possess various types of localized solutions such as pulse-like localized solutions and front-like ones have been proposed while we omit the detail and merely refer to books ([6], [5]). To understand the dynamics of such solutions, reaction-difusion systems have been studied under various situations such as one or higher dimensional spaces, bounded or unbouded domains, and the Neumann boundary conditions or the Dirichlet ones according to considered problems. In fact, the dynamics solutions drastically change depending on the considered situations.

In this talk, we consider farely general types of reaction-diffusion systems

$$\boldsymbol{u}_t = D\boldsymbol{u}_{xx} + F(\boldsymbol{u}),\ t > 0,\ x \in \boldsymbol{R}_+, \tag{1}$$

where $\boldsymbol{R}_+ := [0,\infty)$, $\boldsymbol{u} \in \boldsymbol{R}^N$, $D := diag\{d_1, \cdots, d_N)$ and $F : \boldsymbol{R}^N \to \boldsymbol{R}^N$ is a sufficiently smooth function.

First we consider the problem (1) on $\boldsymbol{R}$

$$\boldsymbol{u}_t = D\boldsymbol{u}_{xx} + F(\boldsymbol{u}),\ t > 0,\ x \in \boldsymbol{R}, \tag{2}$$

and assume several conditions for (2) as follows:

A1) There exists a stable symmetric stationary solution, say $S(x)$ satisfying $S(x) \to e^{-\alpha|x|}\boldsymbol{a}$ as $|x| \to \infty$ for $\alpha > 0$ and $\boldsymbol{a} \in \boldsymbol{R}^N$.

Let $L := \partial_{xx} + F'(S(x))$, the linearized operator of (2) with respect to $S(x)$.

A2) The spectral set $\sigma(L)$ of L is given $\sigma(L) = \sigma_0 \cup \sigma_1$, where $\sigma_0 := \{0\}$ and $\sigma_1 \subset \{Re\lambda < -\gamma_0\}$ for $\gamma_0 > 0$. Moreover, 0 is a simple eigenvalue of L.

Then there exists eigenfunction $\phi^*(x)$ of the adjoint operator L^* of L satisfying $L^*\phi^* = 0$ and $\phi^*(x) \to e^{-\alpha x}\boldsymbol{a}^*$ as $x \to +\infty$ for $\boldsymbol{a}^* \in \boldsymbol{R}^N$. Note that we can take $\phi^*(x)$ as an odd function and by the normalization $\langle\, S_x, \phi^* \,\rangle_{L^2} = 1$, $\phi^*(x)$ is uniquely determined.

Next coming back the original problem (1) on the half line $\boldsymbol{R}_+$. We impose the boundary condition

$$\boldsymbol{u}_x = \beta\boldsymbol{u}, \ x = 0. \tag{3}$$

Then we have

Theorem 1 *Assume A1) and A2). If the initial data* $\boldsymbol{u}(0, x)$ *is sufficiently close to* $S(x - l_0)$ *for* $l_0 >> 1$*, then the solution* $\boldsymbol{u}(t, x)$ *of (1) remains close to*

$$\boldsymbol{u}(t, x) = S(x - l(t)) + O(e^{-\alpha l(t)})$$

as long as $l(t) > l^*$ *for* $l^* >> 1$. $l(t)$ *satisfies*

$$\frac{dl}{dt} = \frac{2\alpha(\alpha - \beta)}{\alpha + \beta} M_0 e^{-2\alpha l}(1 + O(e^{-\alpha l})),$$

where $M_0 := \langle\, D\boldsymbol{a}, \boldsymbol{a}^* \,\rangle$.

In this talk, we will mention more precise analysis about the problems.

参考文献

[1] J. Carr and R. L. Pego, Metastable patterns in solutions of $u_t = \varepsilon^2 u_{xx} + f(u)$, Comm. Pure Appl. Math., 42 (1989), 523-576.

[2] S.-I. Ei, The motion of weakly interacting pulses in reaction-diffusion systems, J.D.D.E. 14(1) (2002), 85-137.

[3] G. Fusco and J. Hale, Slow motion manifold, dormant instability and singular perturbations, J. Dynamics and Differential Equations, 1 (1989), 75-94.

[4] K. Kawasaki and T. Ohta, Kink dynamics in one-dimensional nonlinear systems, Physica 116A (1982), 573-593.

[5] , J. M. Murray, Mathematical biology, Springer-Verlag, N. Y. 1989.

[6] Y. Nishiura, Far-From-Equilibrium Dynamics (Translations of Mathematical Monographs), AMS, 2002.