TSUDA COLLEGE MINI-WORKSHOP

NUMBER THEORY AND PHYSICS AT THE CROSSROADS

AUGUST 4–6, 2008

ABSTRACTS

Yasuhiro Goto (Hokkaido University of Education at Hakodate)

Zeta-functions and L-series of some Calabi–Yau threefolds

We discuss the L-series of some K3-fibered Calabi-Yau threefolds over the rationals. Our threefolds are defined by resolving singularities of weighted diagonal and quasi-diagonal threefolds. We describe their exceptional divisors, compute the L-series and look into the motivic modularity in several cases. This is part of my joint work with R. Kloosterman and N. Yui.

Bill (Jerome) Hoffman (Louisiana State University)

L-functions and ℓ-adic representations for noncongruence subgroups

Let $S_k(\Gamma)$ be the space of weight k cusp forms for a subgroup Γ of SL(2, **Z**). When Γ is a congruence subgroup, Deligne constructed a compatible system of l-adic representations ρ_l associated to this space. The L-function of this, $L(s, \rho_l)$, is expressible as a product of L-functions $L(s, f_j)$ for newforms f_j attached to congruence subgroups of SL(2, **Z**). A. J. Scholl constructed analogous representations for Γ an arbitrary, not necessarily congruence, subgroup. Their properties are strikingly different from the congruence case. Nonetheless, these representations are motivic, and general philosophy predicts they are automorphic, but now possibly for reductive groups other than $\mathrm{GL}(2)_{/\mathbf{Q}}$. We give new examples where this automorphic property can be proved, or at least experimental evidence for it. The weight $k = 3$ and the examples show that $L(s, \rho_l)$ is a product of L-functions for classical newforms, with base-extension and twist by Hecke characters. We discuss general conjectures relating these to automorphic representations for orthogonal groups in four variables.

Shinobu Hosono (University of Tokyo)

An overview for holomorphic anomaly equations II

In this talk, I will continue my talk presented at Tsuda College workshop 2005 under the same title. In the talk 2005, I tried to understand the structure of the BCOV (Bershadsky, Cecotti, Ooguri, and Vafa) holomorphic anomaly equation looking precisely the special Kaehler geometry on the moduli space of a Calabi-Yau threefold. This time, I will continue that viewpoint, and will summarize the developments made by physicists after that. Also, I will probably show some explicit predictions for Gromov-Witten invariants which come from the BCOV holomorphic anomaly equation. These predictions should be interesting in its own right.

Masao Jinzenji (Hokkaido University)

Virtual Structure Constants as Intersection Numbers of Moduli Space of Polynomial Maps with Two Marked Points

In this talk, we derive B-model-like structure constants used in mirror computation, by using localization computation applied to moduli space of polynomial maps from $\mathbf{CP}^1$ with two marked points to complex projective space. We apply this technique to non-nef local geometry $O(1)+O(-3) \to \mathbf{CP}^1$ and realize mirror computation without using Birkhoff factorization.

Masumi Kawasaki (Kaijo High School)

Weierstrass points and gap sequences on certain cyclic branched coverings of the projective plane

Weierstrass points play important roles in the study of the aspects of meromorphic functions on the non-singular algebraic curves. In this talk, we investigate Weierstrass points and Weierstrass gap sequences on cyclic coverings which are branched at four or five points of the projective line. In particular, resultant and sub-resultant are useful for our study.

Kenichiro Kimura (Tsukuba University)

On mixed elliptic motives

I will talk about an attempt to generalize the construction of Bloch-Kriz of mixed Tate motives to mixed elliptic motives.

Masato Kuwata (Chuo University)

Twists of elliptic L-functions and rational points on generalized Kummer varieties

In order to prove the vanishing of critical values of the twists of L-functions of elliptic curves by Dirichlet characters, we need to find points on the elliptic curve defined over the extension corresponding to the Dirichlet character. In the cases of cubic and quartic twists, this problem is reduced to find rational points on a certain K3 surfaces. We explain how this is done, and we discuss possible generalization towards the quintic case, where the associated variety is four dimensional.

Shoetsu Ogata (Tohoku University)

On multiplication of ample and nef divisors on toric surfaces

Let A be an ample line bundle and B a nef line bundle on a toric surface. Then the multiplication map of global sections

$$\Gamma(A) \otimes \Gamma(B) \to \Gamma(A \otimes B)$$

is surjective. As a consequence, we can show that the surjectivity of the multiplication map of global sections of the anti-canonical line bundle and an ample line bundle on a Gorenstein toric 3-fold with the nef anti-canonical divisor.

Yasuo Ohno (Kinki University)

Two-one formula for multiple zeta-star values

Euler, the father of multiple zeta values, mainly treated non-strict multiple zeta values (multiple zeta-star values or MZSVs for short) in his article. The Q-algebras spanned by strict (ordinary) multiple zeta values (MZVs) and MZSVs are the same to each other. I am planning to review some of known relations among MZVs and MZSVs and introduce our new identities and prospects which are proper to MZSVs.

Tsunekazu Saito (Kyushu University)

The dimension of spaces of finite modular forms and actions of the Hecke algebra

Finite upper half-planes over finite fields are defined as finite analogues of the Poincare upper half-plane. A. Terras et al. constructed finite Eisenstein series on finite upper half-planes and proved these series satisfy a certain invariance property under unitary group actions. I determine the dimension of spaces of finite modular forms by constructing the kernel function of these spaces. As a special case, this gives a formula for the dimension of Terras's space containing finite Eisenstein series. I also give a trace formula of actions of the Hecke algebra.

Fumio Sakai (Saitama University)

Weierstrass points and S_4 Geometry on Picard curves

A trigonal cyclic quartic curve is called a Picard curve. Let t denote the number of weight two Weierstrass points on a Picard curve C. We have three cases $t = 1, 4, 7$. M. Kawasaki, in his thesis, found that the locus with $t = 4$ in the 2-parameter family of Picard curves is a singular plane curve of degree 18 and that the locus with $t = 7$ corresponds to the 24 nodes. It is known that the moduli space of Picard curves is described as a S_4 quotient of the parameter space. In this talk, we further study the properties of the image of the above curve in the moduli space.

Yukinobu Toda (University of Tokyo)

Limit stable objects on Calabi-Yau 3-folds

The purpose of this talk is to introduce the notion of limit stability on the category of perverse coherent sheaves on Calabi-Yau 3-folds, and construct new enumerative invariants of curves via limit stable objects. I will show that such invariants are generalizations of stable pair invariants introduced by Pandharipande and Thomas. I will show that investigations of wall-crossing phenomena of limit stable objects lead to the solution of the rationality conjecture of stable pair invariants.

Kazuki Utsumi (Hiroshima University)

On the structure of certain K3 surfaces

I will explain the structure theorem for elliptic K3 surfaces with two singular fibers of type both I_4^*. These K3 surfaces are classified into two classes. One of classes consists of surfaces obtained as double covering of elliptic Kummer surfaces with singular fiber of type I_4^*. The other consists of Kummer surfaces of product type. Moreover, I will show that any K3 surfaces which belong to the former admit Shioda–Inose structure, and others do not.

Yifan Yang (National Chiao Tung University)

Sp_4 modularity of Picard–Fuchs differential equations for Calabi–Yau threefolds

In an earlier work, we showed that if the Picard-Fuchs differential equation of a one-parameter family of Calabi-Yau threefolds is hypergeometric, then its monodromy group is contained in a certain congruence subgroup of $Sp(4, Z)$. This naturally leads to the question whether these differential equations are related to Siegel modular forms. In this talk, we will address this modularity question.

Masahiko Yoshinaga (Kobe University)

Periods and computational complexity of real numbers

The periods, introduced by Kontsevich and Zagier, form a class of complex numbers which contains all algebraic numbers and several transcendental quantities. Little has been known about qualitative properties of periods. In this talk, we compare the periods with hierarchy of real numbers induced from computational complexities. As an application, we exhibit a computable real number which is not a period.

Noriko Yui (Queen's University)

Number Theory and Physics at the Crossroads

Modular forms have long played a key role in the theory of numbers, including most famously the proof of Fermat's Last Theorem. Through its quest to unify the spectacularly successful theories of quantum mechanics and general relativity, string theory has long suggested deep connections between branches of mathematics such as topology, geometry, representation theory, and combinatorics. Less well known are the emerging connections between string theory and number theory. In this talk, I will mention one such example, that is, mirror symmetry and modular forms.

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yui@fields.utoronto.ca, or cntp@intlpress.com

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