The 3rd MSJ-SI
The Mathematical Society of Japan, Seasonal Institute

Development of Galois-Teichmüller Theory and Anabelian Geometry

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Arithmetic questions on $\pi_1(\mathbf{P}^1 - \{0, 1, \infty\})$ *at* p

For an odd prime p, let Π_p (resp. $\bar{\Pi}_p$) denote the quotient of the algebraic fundamental group of $X = \mathbf{P}^1 - \{0, 1, \infty\}$ over $\bar{\mathbf{Q}}_p$ (resp. $\bar{\mathbf{F}}_p$) defined by the condition: the ramification indices above $0, 1, \infty$ are not divisible by p. Let $\Pi_p^0 = \bar{\Pi}^0{}_p$ denote the Galois group of the tower of modular curves $\{X(2N)/X\}_{N \not\equiv 0(p)}$ of level $2N$ over these fields under the identification $X = X(2)$. Look at the canonical surjective homomorphisms $f : \Pi_p \to \bar{\Pi}_p, \quad g : \bar{\Pi}_p \to \Pi_p^0$. Then we see that (i) the kernel of $g \circ f$ is generated by p conjugacy classes, (ii) that of g is generated by $(p+1)/2$ conjugacy classes essentially coming from $(p-1)/2$ supersingular Frobenius elements. These follow easily from our old work on the connections between modular curves over $\mathbf{F}_{p^2}$ and the modular groups over $\mathbf{Z}[1/p]$, which we shall first briefly review. We then go on to discuss some basic (mostly open) questions related to these conjugacy classes and the kernels.

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