$X := \{a_1, \dots, a_m\}$, $a_i \in \mathbb{R}^s$, WEAK DEFINITION BY TEST FUNCTIONS f,

multivariate spline

$$\int_{\mathbb{R}^n} f(v) T_X(v) dv = \int_{\mathbb{R}_+^m} f(\sum_{i=1}^m t_i a_i) dt_1 \dots dt_m,$$

box spline

$$\int_{\mathbb{R}^n} f(v) B_X(v) dv = \int_{[0,1]^m} f(\sum_{i=1}^m t_i a_i) dt_1 \dots dt_m,$$

partition function $a_i \in \mathbb{Z}^s$

$$\sum_{v \in \mathbb{Z}^n} f(v) P_X(v) = \sum_{t_i \in \mathbb{N}} f(\sum_{i=1}^m t_i a_i),$$

TWO SPACES OF FUNCTIONS $D(X), DM(X)$.

For a vector v denote by D_v the usual directional derivative
∇_v the usual difference operator

$$\nabla_v f(x) := f(x) - f(x - v)$$

For a list Y of vectors denote by

$$D_Y = \prod_{a \in Y} D_a, \quad \nabla_Y = \prod_{a \in Y} \nabla_a.$$

$D(X) = \{p : \mathbb{R}^s \to \mathbb{R} \mid D_Y p = 0, \ Y$ runs over all cocircuits in $X\}$.

$DM(X) = \{p : \mathbb{Z}^s \to \mathbb{Z} \mid \nabla_Y p = 0, \ Y$ runs over all cocircuits in $X\}$.