ABSTRACT

Plenary Talks

Shouhei Honda (Tohoku Univ.)
Ricci curvature and Weyl's law

In 1911, H. Weyl provided an asymptotic behavior of eigenfunctions λ_i (counted with the multiplicities) of the Dirichlet Laplacian $-\Delta_\Omega$ on a bounded domain Ω in a Euclidean space $\mathbb{R}^n$ of dimension at most 3, in terms of the size of the domain, namely

$$\lim_{\lambda\to\infty} \frac{N(\lambda)}{\lambda^{n/2}} = \frac{\omega_n}{(2\pi)^n}\mathcal{L}^n(\Omega),$$

where $N(\lambda)$ is the counting function defined by $N(\lambda) = \sharp\{i \in \mathbb{Z}_{\geq 0} | \lambda_i \leq \lambda\}$ and ω_n denotes the volume of a unit ball in $\mathbb{R}^n$. This result was later on extended to higher dimensional (not necessarily flat) *smooth* spaces, including closed Riemannian manifolds (M^n, g). It is known as *Weyl's law.*

On the other hand, based on a joint work with Luigi Ambrosio (Scuola Normale Superiore) and David Tewodrose (Nantes University), it was conjectured that such Weyl's law holds even for *nonsmooth* spaces with Ricci curvature bounded below. The main purpose of the talk provides counterexamples to this conjecture. In the constructions, we can see a new intersection between Gromov–Hausdorff collapsing geometry and subRiemannian geometry. This is a joint work with Xianzhe Dai (UC Santa Babara), Jiayin Pan (UC Santa Cruz) and Guofang Wei (UC Santa Babara).

Jinsung Park (KIAS)
Renormalized volume and Liouville action

The renormalized volume was mathematically defined for conformally compact Einstein manifolds by Robin Graham. This invariant had been introduced and extensively studied in the context of AdS-CFT correspondence by mathematical physics community. For a 3-dimensional convex cocompact hyperbolic manifold M, the renormalized volume is an finite real valued invariant of M if the renormalization process is defined using the hyperbolic metric over conformal boundary ∂M. Hence, the renormalized volume can be considered as a function defined on the deformation space of convex cocompact hyperbolic structures. From mathematical physics results, the renormalized volume is expected to be related to a quantity from conformal boundary ∂M. This quantity called Liouville action was mathematically defined by Takhtajan and Zograf. In this talk, some basic results on the renormalized volume and its relation with the Liouville action will be explained in the context of 2 and 3 dimensional hyperbolic manifolds. Some recent corresponding results for renormalized volume and the universal Liouville action of Weil–Petersson curves will be also explained.

Algebra Session

Yuri Yatagawa (Tokyo Inst. Tech.)
Ramification theory and characteristic cycles of rank one sheaves

The characteristic cycles for constructible sheaves on smooth varieties are algebraic cycles introduced by Beilinson and Saito in 2016. They are defined to be unique algebraic cycles on the cotangent bundle satisfying the Milnor formula, which computes the total dimensions of vanishing cycles, and compute the Euler characteristics of the sheaves by the index formula. In this talk, we consider computations of characteristic cycles in terms of ramification theories of the sheaves. When the varieties are curves, such are already known, and the index formula deduces the classical Grothendieck–Ogg–Shafarevich formula. We restrict the sheaves to be smooth of rank one, which means the sheaves correspond to characters of abelianized fundamental groups, and we introduce recent developments for computations of characteristic cycles with ramification theories of the sheaves.

Byeong-Kweon Oh (Seoul National Univ.)
Recoverable quadratic forms

A (positive definite and integral) quadratic form f is called recoverable if any quadratic form representing all proper subforms of f also represents f itself. If there is a quadratic form F representing all subforms of f, except for f itself, then f is called irrecoverable and F is called an *isolation* of f (from its subforms).

In this talk, we find some (ir)recoverable quadratic forms and various isolations of irrecoverable quadratic forms.

Osamu Iyama (The Univ. of Tokyo)
Tilting theory via g-fans in real Grothendieck groups

The notion of tilting complexes is basic to study the structure of derived categories of rings. The class of silting complexes complements the class of tilting objects from a point of view of mutation. For a finite dimensional algebra A over a field, 2-term silting complexes of A give rise to a simplicial complex (called the *g-simplicial complex*) and a nonsingular fan in the real Grothendieck group of A (called the *g-fan*). For example, the g-fan of a preprojective algebra is the Coxeter fan, and the g-fan of a Jacobian algebra of a certain quiver with potential is the g-fan of the corresponding cluster algebra. The g-fan of A is a useful combinatorial invariant which has a lot of information about representation theory of A, and therefore satisfies many nice properties. For example, the h-vector and the Dehn–Sommerville equation of the g-simplicial complex has a tilting theoretic interpretation. Also the g-fan of A is complete if and only if A has only finitely many 2-term silting complexes up to isomorphism. One of the basic problems is to classify complete g-fans. We give an answer for rank 2 case by showing that complete g-fans of rank 2 are precisely complete *sign-coherent* fans of rank 2. As a by-product, for each positive integer N, we gives a finite dimensional algebra A of rank 2 such that the Hasse quiver of the poset of 2-term silting complexes of A has precisely N connected components. This talk is based on a series of joint works with T. Aoki, A. Higashitani, R. Kase and Y. Mizuno (arXiv:2203.15213, 2301.01498).

Jinhyung Park (KAIST)
Syzygies of secant varieties of smooth projective curves

Exploring the interplay between the geometric properties of algebraic varieties and the algebraic properties of the equations defining algebraic varieties is an important subject in algebraic geometry. The case of smooth projective curves of large degree is fairly well understood. Two major results are Green's $(2g+1+p)$-theorem and the proof of the gonality conjecture by Ein–Lazarsfeld and Rathmann. On the other hand, there has been a great deal of research on secant varieties of projective varieties in the last three decades particularly because some results on secant varieties have found some applications to algebraic statistics and algebraic complexity theory. In this talk, I report recent progress on syzygies of secant varieties of smooth projective curves obtained by joint work with Lawrence Ein and Wenbo Niu and joint work with Junho Choe and Sijong Kwak. In the former, we extend Green's theorem to secant varieties of smooth projective curves by confirming Sidman–Vermeire's conjecture, and in the latter, we prove a generalization of the gonality conjecture: the gonality sequence of a smooth projective curve completely determines the shape of the minimal free resolutions of secant varieties of the curve of large degree. Our results show that there is a "matryoshka structure" among secant varieties of a smooth projective curve.

Geometry and Topology Session

Mayuko Yamashita (Kyoto Univ.)

Algebraic topology and quantum field theories

Recently, there has been a growing interest in the relations between algebraic topology and theoretical physics. Algebraic topology is used to classify physical systems, and it can be a very powerful tool to analyze physical problems in purely mathematical ways. Also, physically motivated conjectures has lead to many interesting developments in homotopy theory. In this talk, I explain these ideas and some of my related works, where we use homotopy theory to study anomalies in string theory.

In my joint work with Yuji Tachikawa (IPMU, Physics) in 2021, we showed the absence of anomaly in heterotic string theory using homotopy theory. Our strategy is to use the Stolz–Teichner conjecture, which is one of the big conjectures connecting homotopy theory and physics, to translate the physics problem into a purely mathematical problem and solve it mathematically. Also, I explain the recent update on that story, where we study the secondary morphism induced from the above vanishing. It turns out that this morphism is closely related to Anderson self-duality of topological modular forms, which had only been understood in a purely homotopy theoretical way. This leads us to new conjectures relating vertex operator algebras and homotopy theory.

Jungsoo Kang (Seoul National Univ.)

A Floer–Gysin exact sequence for prequantization bundles

In this talk, I will discuss the Rabinowitz Floer homology (in short, RFH) of prequantization bundles. Prequantization bundles are circle bundles over closed integral symplectic manifolds which have canonical contact structures. One natural setting in which RFH for a prequantization bundle is defined is when it admits a Liouville filling. I will explain the construction of a Gysin-type long exact sequence relating RFH of a Liouville filling of a prequantization bundle and the quantum homology of the base manifold. Several applications of this exact sequence will also be discussed. On the other hand, prequantization bundles carry natural (non-exact) symplectic fillings, namely the associated complex line bundles. In this case, the chain complex of RFH is more sophisticated than the one in the Liouville setting. I will introduce filtrations for the chain complex of RFH for complex line bundles and present computational results. This is based on joint work with Peter Albers, Joonghyun Bae, and Sungho Kim.

Kimihiko Motegi (Nihon Univ.)

Dehn filling and knot group: the Property P conjecture and beyond

For a given knot K in the 3–sphere, removing its tubular neighborhood to obtain the exterior $E(K)$, which has a torus boundary. Gluing back a solid torus to $E(K)$ along their boundaries so that a meridian of the solid torus represents a slope r (i.e., an unoriented isotopy classes of simple loops on $\partial E(K)$), we obtain a closed 3–manifold. This operation introduced by Max Dehn is called r–Dehn surgery on K, and the gluing part is called r–Dehn filling of $E(K)$. Slopes are parametrized by rational numbers. The Property P conjecture, settled by Kronheimer and Mrowka, asserts that every non-trivial knot satisfies Property P: for any slope $r \in \mathbb{Q}$, there exists a non-trivial element $g \in \pi_1(E(K))$ which remains non-trivial after r–Dehn filling. (This is a paraphrase of the original Property P.) In this talk we discuss "extension", "enhancement" and "refinement" of Property P from group theoretic perspective, and demonstrate somewhat surprising flexibilities of Dehn filling trivializations. This is joint work with Tetsuya Ito (Kyoto University) and Masakazu Teragaito (Hiroshima University).

JungHwan Park (KAIST)
Seifert surfaces in the 4-ball

We answer a question of Livingston from 1982 by producing Seifert surfaces of the same genus for a knot in the 3-sphere that do not become isotopic when their interiors are pushed into the 4-ball. We give examples where the surfaces are not topologically isotopic in the 4-ball, as well as examples that are topologically but not smoothly isotopic. These latter surfaces are distinguished by their associated cobordism maps on Khovanov homology, and our calculations demonstrate the stability and computability of these maps under certain satellite operations. This is joint work with Kyle Hayden, Seungwon Kim, Maggie Miller, and Isaac Sundberg.

Analysis Session

Ryo Takada (The Univ. of Tokyo)
Large time behavior of global solutions to the Navier–Stokes equations with the Coriolis force

In this talk, we consider the large time behavior of global solutions for the initial value problem of the incompressible Navier–Stokes equations with the Coriolis force in $\mathbb{R}^3$. We shall show the L^p temporal decay estimates with the dispersion effect of the Coriolis force for global solutions when the initial velocity u_0 belongs to $L^1(\mathbb{R}^3)$. Furthermore, under the additional assumption that $|x|u_0 \in L^1(\mathbb{R}^3)$, we establish the large time asymptotics of global solutions behaving like the first-order spatial derivatives of the integral kernel of the corresponding linear solution. This talk is based on the joint work with Takanari Egashira (Kyushu University).

Seick Kim (Yonsei Univ.)
On Schauder type estimates for elliptic and parabolic PDEs in double divergence form

We consider the elliptic operator L^* of the form

$$L^*u = \sum_{i,j=1}^{d} D_{ij}(a^{ij}u) - \sum_{i=1}^{d} D_i(b^i u) + cu,$$

The operator L^* is called double divergence form operator and is the formal adjoint of the elliptic operator of non-divergence form L given by

$$Lv = \sum_{i,j=1}^{d} a^{ij} D_{ij}v + \sum_{i=1}^{d} b^i D_i u + cu.$$

An important example of a double divergence form equation is the stationary Kolmogorov equation for invariant measures of a diffusion process.

We are concerned with regularity of weak solutions of $L^*u = 0$ and show that Schauder type estimates are available when the coefficients a^{ij} are of Dini mean oscillation and b^i and

c belong to certain Lebesgue classes. We will also discuss some applications and parabolic counterparts.

Yasuhito Miyamoto (The Univ. of Tokyo)

Structure of radial solutions for supercritical elliptic equations

We are interested in a bifurcation structure of positive solutions for supercritical elliptic Dirichlet problems in a ball, i.e., $\Delta u + \lambda f(u) = 0$ in B. Since a growth rate of a nonlinear term is larger than the critical Sobolev growth, the corresponding energy functional is not well-defined in H^1. It is difficult to use variational approaches, and a solution structure in a general domain remains not so clear as subcritical cases. In this talk we restrict a domain to a ball. Using ODE approaches, we can derive detailed information about a solution structure. Singular solutions play a crucial role in this study, and hence those are studied in detail.

Moon-Jin Kang (KAIST)

Stability of small BV solutions to compressible Euler equations in the class of inviscid limits from Navier–Stokes

The convex integration shows that the compressible Euler system in multi-D is ill-posed in the class of entropy solutions, especially, non-uniqueness of entropy solutions containing a shock wave. Recently, for the 1D isentropic case, we showed the uniqueness and stability of entropy solutions of small BV in the class of vanishing physical viscosity limits, that is, inviscid limits from the associated Navier–Stokes system. These results use the so-called 'a-contraction with shifts' method for handling the uniform stability of a viscous shock. In this talk, I will explain on the method of a-contraction with shifts for a single viscous shock, and extension of the method to more general situations for two shocks and small BV solutions. This is joint work with Geng Chen and Alexis Vasseur.

Probability Theory and Applied Mathematics Session

Takahito Kashiwabara (The Univ. of Tokyo)

Error estimates of the finite element method in smooth domains

The finite element method (FEM) is one of the most commonly-used numerical methods to approximate solutions of partial differential equations. One of its advantages, often mentioned in textbooks etc., is the applicability to a general domain Ω with a curved boundary. In fact, it is true that the FEM does not require cartesian grids. However, since it is impossible to exactly partition such Ω into triangulations with flat faces, the approximate problem is actually constructed on an approximated domain Ω_h. Therefore, from the mathematical viewpoint, we need to address "domain perturbation errors" caused by the discrepancy between Ω and Ω_h, a general theory of which seems less developed. In this talk, we present a strategy to deal with domain perturbation errors arising in FEM, which is applied to some fluid problems, to maximum-norm error estimates, and to a generalized Robin boundary condition involving the Laplace–Beltrami operator.

Un Cig Ji (Chungbuk National Univ.)

Analytic representations of CAP operators canonically associated with random variables

Let X be a random variable with finite moments of all orders. By the three-term recurrence relation of the orthogonal polynomials associated with the distribution of X, the random variable X as a multiplication operator is represented as a sum of three linear operators called creation, annihilation and preservation operators (CAP-operators). In this talk, we first discuss analytic representations of the CAP operators in terms of differential operators with polynomial coefficients. This approach extends the usual Boson quantum mechanics corresponding to the Gaussian measure to the quantum mechanics associated with random variables. Secondly, we introduce the notion of finite type random variable and characterize type-2 and type-3 real-valued random variables. We also discuss a necessary condition for a random variable to be of finite. This allows to prove that the Beta and the uniform distributions are of infinite type. This talk is based on a joint work with L. Accardi, A. Ebang Ella and Y. G. Lu.

Masanori Hino (Kyoto Univ.)

Martingale dimensions of diffusion processes associated with Dirichlet forms

The concept of martingale dimension is defined for diffusion processes or more-general stochastic processes, and is interpreted as the multiplicity of filtration. Such a kind of concept was introduced at least in the 1960s. In a typical situation where the process is a solution to a stochastic differential equation on the Euclidean space, the martingale dimension corresponds to the number of independent noises that the process possesses. For diffusion processes on fractal-like sets, however, determining the martingale dimension is a difficult problem. The first result in this direction was obtained in a 1989 paper, where it was shown that the martingale dimension of the Brownian motion of *any* dimensional Sierpinski gasket is one. This may seem counter-intuitive. The relationship between the martingale dimension and other kind of dimensions is not yet fully understood. In this talk we give a survey of developments that followed and discuss the cases of diffusions on general state spaces.

Byungjoon Lee (Catholic Univ. of Korea)

Super-convergence of the Poisson solver with adaptive grid configurations

The Hodge decomposition, that is an important feature of incompressible fluid flows, is orthogonal and the projection taking its incompressible component is therefore stable. The decomposition is implemented by solving the Poisson equation. In order to simulate incompressible fluid flows in a stable manner, it is desired to utilize a Poisson solver that attains the orthogonality of the Hodge decomposition in a discrete level.

When a Poisson solver induces the orthogonality, its associated linear system is necessarily symmetric. With this regard, the symmetric Poisson solvers by Losasso et al. [1, 2] are more advantageous not only to efficiently solving the linear system but also to stably simulating fluid flows than nonsymmetric ones. Their numerical solutions were empirically observed to be first and second order accurate, respectively. One may expect that each of their numerical gradients has convergence order that is one less than that of its numerical solution.

However, we in this work show that super-convergence holds true with both Poisson solvers. Rigorous analysis is presented to prove that the difference is one half, not one between the convergence orders of numerical solution and gradient in both solvers. The analysis is then validated with numerical results. We furthermore show that both Poisson solvers, being symmetric, indeed satisfy the orthogonal property in the discrete level and yield stable implementations of the Hodge decomposition in octree grids.

[1] F.Losasso, F.Gibou, and R.Fedkiw, *Simulating water and smoke with an octree data structure*, ACM SIGGRAPH, 2004.

[2] FRSF.Losasso, R.Fedkiw, and S.Osher, *Spatially adaptive techniques for level set methods and incompressible flow*, Computer and fluid, 2006.