

SUMMARY

1. PEANRY

Yasuyuki KAWAHIGASHI (Univ. of Tokyo)

Topological order, tensor categories and operator algebras

Two-dimensional topological order has attracted much attention in physics recently. It is related to topological quantum computing, for example, which is a target of a great deal of industrial research today. This topic has many mathematical aspects related to different fields in mathematics. In this talk, we present connections to operator algebras based on tensor categories with emphasis on a representation theoretic viewpoint. We first explain a general notion of a tensor category and then relate it to theory of operator algebras, particularly subfactors of the so-called hyperfinite III₁ factor in theory of Jones. With these preliminaries, we explain recent progress on two-dimensional topological order, chiral conformal field theory and tensor networks. Abstract begins.

Inkang KIM(KIAS)

Signature, Toledo invariant, eta invariant, and the representation of surface groups in real symplectic group. In this talk, by using Atiyah-Patodi-Singer index theorem, we obtain a formula for the signature of a flat symplectic vector bundle over a surface with boundary, which is related to the Toledo invariant of a surface group representation in the real symplectic group and the Rho invariant on the boundary. As an application, we obtain a Milnor-Wood type inequality for the signature. In particular, we give a new proof of the Milnor-Wood inequality for the Toledo invariant in the case of closed surfaces and obtain some modified inequalities for the surface with boundary.

2. ALGEBRA

Takehiko YASUDA(Osaka Univ.)

On the isomorphism problem for projective schemes

In this talk, I discuss the isomorphism problem for (quasi-)projective schemes over the field of algebraic numbers. This asks whether there is an algorithm to decide whether two projective schemes are isomorphic. The main result is decidability of isomorphism in the cases of (1) one-dimensional projective schemes, (2) one-dimensional reduced quasi-projective schemes, (3) smooth irreducible projective varieties with canonical or anti-canonical divisor being big, and (4) K3 surfaces with finite automorphism groups. As a closed related problem, I also discuss decidability of positivity properties of invertible sheaves.

Sanghoon BAEK (KAIST)

Rationality and complexity for spaces of torsors under semisimple groups

Consider a generically free representation V of a semisimple group G over the complex numbers C . Then, there exists a G -torsor over the quotient U/G (called the classifying space of G) for an open subset U of V . Such a G -torsor is called versal as for every field extension K/C , the K -points of the classifying space parametrize all G -torsors over K . In this talk, we shall discuss two wide open problems: the rationality problem of the classifying space and the problem of finding the minimum dimension of the classifying space of G through all versal G -torsors. Also, we present some recent results on the unramified cohomology of the classifying space detecting the rationality and the exact values of the minimum dimension of the classifying space of the groups of type B.

Akio TAMAGAWA (Kyoto Univ.)

“Ghosts” in families of abelian varieties with a common isogeny factor (joint work with Anna Cadoret)
 We discuss the following question raised by Rössler and Szamuely. Given a family of abelian varieties over a variety over a field, assume that all the closed fibers have a nontrivial common isogeny factor. Does this imply that the generic fiber has a nontrivial isotrivial isogeny factor? This question can be reformulated in terms of ℓ -adic representations and the difficult case is when the base field is finite. In this setting, we introduce the notion of ghosts in terms of ℓ -adic representations. Ghosts control completely the problem, namely the original question is positive if and only if the ghost is trivial. We prove the triviality of the ghost in various cases and, in general, it is predicted by Zarhin’s microweights conjecture.

Jihun PARK (POSTECH)

Sasaki-Einstein 5-manifolds from an algebro-geometric point of view

We present the state of the art concerning classification of closed simply connected Sasaki-Einstein 5-manifolds from an algebro-geometric point of view.

3. GEOMETRY AND TOPOLOGY

Koichi NAGANO (Univ. of Tsukuba)

Topological regularity theorems for CAT(0) spaces

In this talk, I would like to focus on topological regularity problems of when CAT(0) spaces are homeomorphic to Euclidean spaces. I will give an overview of a study on asymptotic topological regularity of CAT(0) spaces. I will present conditions for CAT(0) spaces of small volume growth to be homeomorphic to Euclidean spaces. I will also report the answer to the question of Gromov for CAT(0) spaces whether there exist CAT(0) topological manifolds other than Euclidean spaces in the remaining 4- dimensional case. The part on the answer to the question of Gromov for CAT(0) spaces is a joint work with Alexander Lytchak (University of Cologne) and Stephan Stadler (Max Planck Institute for Mathematics).

Yunhyung CHO (Sungkyunkwan Univ.)

MONOTONE LAGRANGIANS AND MUTATIONS IN FLAG VARIETIES

Using a Gorenstein Fano toric degeneration, one can construct a Lagrangian torus fibration on a smooth Fano variety X . Such a fibration has a unique central fiber which becomes a monotone Lagrangian torus and its Maurer- Cartan space contains a complex torus equipped with a Laurent polynomial $W : (\mathbf{C}^*)^n \rightarrow \mathbf{C}$ called a Fukaya-Oh-Ohta-Ono’s disc potential. When a degeneration is good, the potential function W coincides with the Landau-Ginzburg superpotential of X which captures the multiplicative structure of the quantum cohomology ring of X . In this talk, I will explain this picture for flag varieties. Also I will explain how those potential functions are related by cluster mutations.

Hiroshi OHTA (Nagoya Univ.)

Analytic aspect of Fukaya category

For a symplectic manifold (X, ω) we have a filtered A^∞ category called Fukaya category, whose objects are Lagrangian submanifolds. I will discuss some analytic aspect of certain meromorphic connection arising from deformations of Fukaya categories.

Cheol-Hyun CHO (SNU)

Berglund-Hübsch conjecture for invertible curve singularities

Given one of the polynomials $W = xp + yq, xp + xyq, xp y + xyq$ and a diagonal symmetry group G , Berglund-Hübsch mirror symmetry conjectures that the symplectic geometry of W is related to the complex geometry of transpose polynomial W^T with the transpose symmetry group GT . We first define a new Fukaya category of the Landau-Ginzburg orbifold (W, G) , from the wrapped Fukaya category of its Milnor fiber together with monodromy information. We formulate and prove a full version

of BH homological mirror symmetry conjecture in these cases. Furthermore, for ADE singularities, Auslander-Reiten quivers for indecomposable matrix factorizations were known from 80's, and we find the corresponding Lagrangians as well as surgery exact triangles.

4. ANALYSIS

Neal BEZ (Saitama Univ.)

The nonlinear Brascamp-Lieb inequality

The Brascamp-Lieb inequality provides a natural generalisation of the multilinear Holder inequality, the Loomis-Whitney inequality and the Young convolution inequality. For a number of reasons, including applications to harmonic analysis and PDE, it is of interest to establish a nonlinear variant of the Brascamp-Lieb inequality. I will explain some recent work with Jonathan Bennett, Stefan Buschenhenke, Michael Cowling and Taryn Flock, where we establish such a nonlinear Brascamp-Lieb inequality in full generality.

Soonsik KWON (KAIST)

Blow-up dynamics of the self-dual Chern-Simons-Schrödinger equations

This abstract and talk is based on joint works with Kihyun Kim and Sung-Jin Oh [11, 12, 13]. We study a quantitative description of the blow-up dynamics of solutions near the vortex solutions Q to the self-dual Chern-Simons-Schrödinger equation under equivariant symmetry. In this abstract, we first introduce the self-dual Chern-Simons-Schrödinger equations and then announce our results on their blow-up dynamics. All the statements of results are informal here. For more precise statements and details, we refer to original papers.

Takayoshi OGAWA (Tohoku Univ.)

End-point maximal regularity and free boundary problem of the Navier-Stokes equations

We consider maximal regularity for the initial boundary value problem of the heat and the Stokes equations. Maximal regularity generally ensures that regularity of solutions for parabolic partial differential equations can be obtained up to regularity for the external forces. Theory of maximal regularity is well developed in the framework of UMD (unconditional martingale differences) Banach spaces and the equivalent characterization for the well-posedness is established for not only the external force but for the initial and boundary traces. Since UMD Banach spaces are necessarily reflexive, maximal regularity in non-reflexive Banach spaces is not clear in general. Here we consider maximal time- L^1 -regularity over a non-reflexive Banach space with inhomogeneous boundary condition in the half Euclidean space and apply it to the well-posedness issue for free boundary problems of the incompressible Navier-Stokes equations. This talk is based on a joint work with Prof. Senjo Shimizu (Kyoto University).

Kyungkeun KANG (Yonsei Univ.)

Local regularity of Stokes system and Navier-Stokes equations near boundary

We review some results about local regularity for the Stokes system and the Navier-Stokes equations near boundary. In a half-space, it was shown in [8] that a solution generated by a compactly supported, Hölder continuous boundary flux from a far may have unbounded normal derivatives near the boundary. These phenomena are mainly due to nonlocal effect caused by singular flux far away from a boundary region under consideration. Such singular solutions turn out to be of finite global energy and they can be also constructed to the Navier-Stokes equations as well (see [9]). It is also possible to construct solutions that are bounded but the gradients of velocity fields are singular in any $L^p, p > 1$ in a local neighborhood near boundary (see [2]). Furthermore, using the construction of singular solutions whose gradients are not in L^2_{loc} , it was proved in [3] that the Caccioppoli's inequality in general fails for both Stokes system and Navier-Stokes equations. The local maximal regularity including boundedness of gradient of velocity was analyzed in [2] as well.

Works are partly collaborated with T.-K. Chang or with B-Lai, C.-C. Lai and T.-P. Tsai.

5. PROBABILITY THEORY AND APPLIED MATHEMATICS

Kenta ISHIMOTO (Kyoto Univ.)

On Flows and Form of Microswimmers

Microswimmers are microscopic self-propelled particles in fluids. In this article, we give an introductory review on the hydrodynamics of microswimmers as an example of a moving boundary problem in fluid dynamics research. In particular, we focus on the dynamics of a microscopic object in a fluid flow and provide a new formula for a chiral active particle such as swimming bacteria.

Ildoo KIM (Korea Univ.)An L_p -theory for second-order stochastic partial differential equations

In this talk, we introduce a short history of L_p -theories for second-order stochastic partial differential equations (SPDEs). Especially, we present existence, uniqueness, and sharp regularity results of solution to the stochastic partial differential equation (SPDE) of the form of

$$du = (a^{ij}(\omega, t, x)u_{x^i x^j} + f)dt + (\sigma^{ik}(\omega, t, x)u_{x^i} + g^k)dw_t^k, u(0, x) = u_0,$$

where $\{w_t^k : k = 1, 2, \dots\}$ is a sequence of independent Brownian motions. It is notable that if the coefficients are independent of the space variable x , then they are allowed to be merely measurable in (ω, t) and can be unbounded and fully degenerate, that is, coefficients a^{ij}, σ^{ik} merely satisfy

$$(\alpha^{ij}(\omega, t))_{d \times d} := \left(a^{ij}(\omega, t) - \frac{1}{2} \sum_{k=1}^{\infty} \sigma^{ik}(\omega, t) \sigma^{jk}(\omega, t) \right) \geq 0.$$

Yuzuru INAHAMA (Kyushu Univ.)

Heat trace asymptotics for equiregular sub-Riemannian manifolds

We discuss a “div-grad type” sub-Laplacian with respect to a smooth measure and its associated heat semigroup on a compact equiregular sub-Riemannian manifold. We prove a short time asymptotic expansion of the heat trace up to any order. Our main result holds true for any smooth measure on the manifold, but it has a spectral geometric meaning when Popp’s measure is considered. Our proof is probabilistic. In particular, we use S. Watanabe’s distributional Malliavin calculus.

Myungjoo KANG (SNU)

Some applications using Deep Learning Techniques

Since 1990, many mathematicians studied the image processing based on partial differential equations and variational methods. Using TV(total variation) and some optimization techniques, there were a lot of improvement in image processing areas. Until deep learning coming out, those were the state of art methods in these areas. But once using deep learning techniques, it turns out that in almost every areas in image processing fields, deep learning is the best method. I will compare the results between the variational image processing techniques and deep learning. Also, I will give some industrial applications using deep learning.