# Towered Arithmetic structures, Splitting primes and Zeta functions

– following history of number theory promoted by curiosity for "odd" structures –

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Each Galois extension $K/k$ of an arithmetic field associates, to each (unramified) prime of the base $k$, a conjugacy class of the Galois group $G$. But this cannot be expected to be even close to being 'reciprocal', unless the extension is infinite-and-'tight', ramifications are restricted, and $G$ is a profinite completion of a discrete group $\Gamma$ whose conjugay classes can be compared with the prime-powers of $k$. In the ideal case the Riemann-type zeta of $k$ would be explained by a Selberg type zeta for $\Gamma$.

My search for 'good towers' had started by findings of such, for function fields $k$ over finite fields, together with $\Gamma$ which, necessarily, was an $(\infty \times p)$-adic discrete group. Supersingular moduli in a generalized sense played a central role in producing as many throughout rational points as possible, thus 'tightening $K/k$'. Yes, an old story; indeed, so old that someone felt it necessary this long-sleeping *immature* beauty be woken up now. Comparison with the number field cases will also be discussed.