Methodology and strategies for mathematical sciences

Hiroshi Fujita (Professor Emeritus, The University of Tokyo)

1. Mathematical science and myself

1.1. Introduction

I had entered the world of mathematics from mathematical physics as a physics student. When I was a graduate student at the Department of Physics of University of Tokyo, Profs. Kunihiko Kodaira and Tosio Kato who became later on, great mathematicians had been young associate professors. Besides them, Profs. Isao Imai and Hidetoshi Takahashi influenced me so intensively that I would devote myself to the mathematical science. I believe that I am awarded the Prize of Kunihiko Kodaira because of my contribution to several problems arising from mathematical sciences via the method of functional analysis.

1.2. Evolution of mathematical science

The term, "Mathematical Science" has started to be used in Japan in the middle of the last century, that is, from the end of the 1950s to the beginning of the 1960s. At the same period, Research Institute of Mathematical Science (RIMS) was founded on account of great leadership due to President Prof. Shokichi Iyanaga who was favorable to collaboration between mathematics and other applied sciences. In such an environment, physics and in particular, the applied analysis based on the mathematical modelling by means of differential equations play a central role. On the other hand, the control theory and mathematical statics were not treated as main subjects in RIMS, and only laboratory of the numerical analysis was founded, but it was too weak to carry out the full-scale computer simulation. This means that RIMS was supposed to be oriented to the pure mathematics, at most to the mathematical science based on the natural science, which had gotten behind the main academic stream from the world. Fortunately, at that period, I had spent my academic carrier in the faculty engineering where I could collaborate jointly with colleagues having the skill of the finite element method in numerical analysis.

With the passage of time, the applied mathematical science was admitted as a variable subject in our academic society so that The Japan Society for Industrial and Applied Mathematics (JSIAM) has been also founded. It is now the time that "mathematical science for understanding" and "engineering mathematics for achievement", which were unfortunately disconnected in mathematical sciences in the last century, become Interdisciplinary to each other. It should be emphasized that even in the engineering mathematics for achievement, understanding of the mathematical structure locally or preferably globally has been nowadays may be regarded as the key with high evaluation. I wish to feel realization of development of the mathematical sciences, and in particular, to devote myself to contribution to it in terms with the method of the functional analysis. From 1959 to 1965, I was an associate professor in the Department of Mechanics of the Faculty of Engineering founded by Prof. Kanich Terasawa, the great pioneer of the mathematical physics in Japan, where I became on intimate terms with Prof. Masatake Mori, the first

President of The Japan Society for Industrial and Applied Mathematical, and Prof. Hideo Kawarada, an expert of finding problems in the world-wide level. Besides them at that period, I had a good opportunity to obtain various knowledge from other authorities in mathematical sciences, which is based on my academic achievement over seventy years.

Tale 1. I really hope that many researchers and instructors acting in this new century will be related to Understanding and Achievement of Mathematical Science in accordance with their own academic characterization and environment, and make further progress with wide perspective.

Tale 2. We may omit it.

Tale 3. In various circumstances under such an influence as mechanical deep learning through AI, I believe that mathematics itself should be developed by including vitality of creativity arising from new revolution in sciences. This is due to H. Poincaré.

2. Expected roles of mathematics in mathematical sciences

2.1. From mathematical science news

Let us consider an expected role of mathematics, for instance in collaboration with the physics as a fundamental subject in mathematical sciences. To this end, I refer "Mathematical Science News" issued by Prof. Iyanaga as a report for the interview to Prof. Takahiko Yamanouchi, an authority in the theoretical physics as well as other guests of the world-wide top stars in mathematical sciences. Profs. Mikio Sato, Hideo Takami and myself were editors of this magazine. Here is a summary of the proposals which has been expected in mathematics from the physical viewpoint.

(i) Opinion of Yamanouchi (conversation with Prof. Yasuo Akizuki)
Mathematicians should recognize new perspectives arising from mathematical sciences, and develop mathematics itself by constructing new areas.

(ii) Opinion of Yamanouchi (conversation with Prof. Yasuo Akizuki)
There is no independent field of mathematical science. The value of the obtained results is evaluated in each field.

(iii) Opinion of Yamanouchi (conversation with Prof.Isao Imai)
Mathematicians are kindly expected, for instance, to explain basic notions of the Lie group. The problem whether it is applicable will be handled by physicians.

(iv) Opinion of Imai
For instance, the framework of field theory in physics should be treated first by physicians, and then mathematicians are expected to construct it as a rigorous theory in mathematics. Concerning computational method, physicians perform it based on the physical inspiration or intuition. On the other hand, we expect that mathematicians will clarify it in a systematic way so that it is mathematically understandable to all scientists.

2.2. Practical mathematical science methodologies

The main task expected in mathematics is to build a model which describes the phenomenon in terms of mathematics. It is necessary to gather all professional concepts in each specific field combined with the mathematical insight for the object. Once a mathematical model has been established, it should be analyzed in such a way whether both efficiency and reliability of the model will be evaluated. Further procedure in the case of successful models and the feedback in the case of necessary improvement are required. As a summary, I propose

Target (for problem setting / purpose awareness),
→ Model (Mathematical formulation)
→ Mathematical analysis (method mobilization)
→ Evaluation / Feedback.

2.3 Consideration of Yamauchi and Imai recommendations

1. Answer to Yanouchi's suggestion

It might be regarded as an encouragement for mathematicians in the sense that requirements of physics will make further progress of mathematics. The result should be evaluated in accordance with values in each field, which may be different from that of mathematics. It would be nice if both evaluations are positive. I believe that the mathematical theory for the quantum mechanics in terms of Schrödinger equation as well as the Hamiltonian system founded by Tosio Kato is one of the most successful example in collaboration with the pure mathematics and theoretical physics. Furthermore, the method of solvability for the initial value problem of Navier–Stokes equations had been established by Tosio Kato and myself by means of operator theory in functional analysis, which made a fundamental breakthrough in the research of the nonlinear PDEs. It should be noticed that final answer to global solvability remains as one of "Millennium Problems" proposed by the Clay Institute in 2000.

2. Answer to Imai's suggestion

It is my pleasure that the Fujita–Kato theory for the functional analytic approach to the Navier–Stokes equations was highly evaluated by Prof. Imai since it is quite understandable even for theoretical physicians. It is necessary for both mathematical scientists and mathematicians to be patient with making an effort to build mutual understanding although they use their own different technical terminologies in communication. For instance, I guess that Prof. Takashi Suzuki, who has recently constructed mathematical oncology in cooperation with doctors, seems to need a special training to communicate with medical doctors. Concerning Imai's suggestion, I believe that the perturbation theory for the Schrödinger equation is a typical example with which theoretical physicians are satisfied, and that a plenty of results have made further developments even within the pure mathematics. Recently, Prof. Mitsuhiro Nakao developed the theory of numerical approximation which may regarded as a successful example based on Tosio Kato's tradition.

2.4 Mathematical phenomena unearthed from real problems

Prof. Kunihiko Kodaira had the opinion that mathematical phenomena are more fundamental than physical phenomena. I was successful to establish a compete result on solvability of the initial value problem on the 2D Navier–Stokes equations. However, I had come across so hard problem on the 3D case so that I reached the blowing-up problem on the nonlinear heat equation. Taking the balance into account between the space dimension and the nonlinearity, I succeeded to find the critical exponent which is nowadays called "Funjita's exponent".

3. Role of mathematics in mathematical sciences for the new century

I am quite sure that "concept and method are the pillars of understanding and achievement". In addition, the criteria for satisfaction and the criteria for performance evaluation must be fully considered.