On Takashi Kumagai, Waseda University,
receiving a Commendation for Science and Technology
by the Minister of Education, Culture, Sports, Science and Technology

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It is my pleasure to report that, earlier this year, Takashi Kumagai received the commendation mentioned in the title for his work on anomalous diffusion phenomena in complex systems. This is richly deserved, not only for his exceptional research achievements, but also for his leadership in our probability research community and beyond.

As I am sure many of you know, Takashi's success has been longstanding. As a result, this is not the first award of his that has been reported on in this venue. Indeed, for his JSPS Prize and Inoue Prize for Science, you can find earlier congratulatory articles written by his seniors Shigeo Kusuoka and Ichiro Shigekawa that give a sense of how Takashi grew into the researcher he is today [9, 10]. Moreover, on the occasion of him being granted the Humboldt Research Award, one his younger colleagues, Naotaka Kajino, gave a more detailed overview of Takashi's work up to 2018, and his contributions to sub-Gaussian heat kernel estimates in particular [6].

This being the case, I will focus here on Takashi's more recent work. For those that do not know me, I have been fortunate to have been one of Takashi's many collaborators, and very much enjoyed working with him over a period of nearly two decades. Certainly, it is not possible to do justice to the full range of his contributions here, and naturally my knowledge of his work has a personal perspective, but nonetheless I hope to give something of a flavour of the general scientific importance of his work.

The topic at the basis of Takashi's award is motivated by problems in physics concerning electron or heat transfer, for example. For context, the behaviour of a diffusion process in a suitably smooth Euclidean domain has been well-understood for a long time now. Indeed, it is classical knowledge in statistical mechanics that the stochastic process known as Brownian motion can be used to describe the motion of individual diffusing particles and Gaussian-type distributions to describe the overall density of particles, and these objects are extremely well-understood mathematically. Providing ever more robust conditions under which such phenomena can be seen, even in the presence of microscopic deformations of the medium, which is to say homogenisation occurs, has been a major area of research in probability and analysis over the last forty years. Additionally, more than forty years ago, physicists noticed that not all media exhibited the same kind of behaviour. In particular, if the irregularity of the medium is suitably high, so-called sub-diffusive behaviour can be seen, whereby particles take longer to reach a given distance than in the Euclidean setting due to traps in the environment. In terms of the mathematical models used to study this phenomena, random walks on percolation clusters at criticality are seen as fundamental examples. Whilst such models remain challenging today, understanding them has inspired great research efforts.

Takashi's work has many strands, but within these are important contributions to both of the areas introduced in the previous paragraph, namely homogenisation and sub-diffusive stochastic processes. To begin with the area of homogenisation, as indicated above, the basic aim is to demonstrate that a stochastic process in some disordered media, often a random walk on a random graph, scales to Brownian motion in $\mathbb{R}^d$. In the case when the individual jumps of the random walk are bounded, conditions for such a result to hold have been relatively well-understood for some time now. However, if the jumps are unbounded, or 'long-range', then Brownian motion is no longer the only natural candidate for a limiting process. Indeed, in the case when the long jumps persist in the limit, then so-called stable-like processes can appear instead. As just one example, consider a random walk on $\mathbb{Z}^d$ that jumps from x to y with probability given by $C_x\omega_{x,y}|x-y|^{-d-\alpha}$, where C_x is a normalising constant depending on x, $\alpha \in (0,2)$ is a fixed constant and the $\{\omega_{x,y}\}_{x,y\in\mathbb{Z}^d,\, x\neq y}$ are independent random weights uniformly bounded away from 0 and ∞. Together with his collaborators, one of Takashi's main contributions in recent years has been to develop homogenisation theory to handle such stochastic processes with long-range jumps. This program has involved adapting analytic tools used to prove heat kernel (transition density) estimates in the Gaussian case to ones suitable for handling stable-like processes, and also building theory suitable for deriving scaling limits – often this involves the connection between stochastic processes and Dirichlet forms. For those interested in further details, see [5] for a survey of some recent results concerning heat kernel estimates in this area and [3, 4] for some indicative homogenisation results.

In terms of sub-diffusive stochastic processes, the mathematical story starts with the study of analysis and probability on deterministic self-similar fractals, such as the Sierpinski gasket. As can be read in the earlier articles in this series [6, 9, 10], Takashi came to this field early on, both in terms of his career and its development. He would soon become one of the leading figures in the area, and, together with other researchers, such as Jun Kigami, establish Japan (and Kyoto in particular) as a centre of international excellence for this kind of research. The techniques developed for studying stochastic processes on fractals have been strengthened sufficiently over the years so that they are now closer to handling the kinds of spaces originally motivating the research, namely random graphs in critical regimes, which are known to exhibit large-scale fractal structure. In his work, Takashi has made vital contributions to this program in various directions: constructing stochastic processes on anomalous spaces, i.e. fractals, both deterministic and random; demonstrating that these arise as the scaling limits of related random walks; and deriving sub-Gaussian heat kernel estimates for these. A summary of this research area by Takashi himself can be found in the article accompanying his 2014 ICM address [7] (alternatively, for a briefer treatment, see the more recent [8]).

As just one significant example that Takashi has worked on recently, one has the two-dimensional uniform spanning tree, which is a model arising naturally in statistical mechanics. Roughly speaking, this is a random connected subgraph of $\mathbb{Z}^2$ that is chosen uniformly from amongst those that contain all the vertices of $\mathbb{Z}^2$ and have no cycles. The spatial constraints that exist lead to strong dependence between different parts of the environment, which make it a challenge to deal with; in fact, the framework for describing its scaling limit, based on what is now called Schramm-Loewner evolution, led to the Fields medal of Werner in 2006. In the joint works [1, 2], a scaling limit for the associated random walk and detailed transition density estimates for that and the limiting process were derived.

Beyond his individual research contributions, the thing that stands out to me about Takashi is his vision. Indeed, Takashi is constantly thinking about how best to advance probability research in Japan. His efforts have inspired numerous meetings and research programs, and at these, and elsewhere, Takashi proactively aims to connect researchers to take projects forward. Moreover, he is very minded about the future, and puts a great deal of energy into providing opportunities for younger researchers to develop their careers. I have certainly benefitted in this way personally. More broadly, such work has no doubt had an intangible impact on the development of probability in Japan.

Often when you receive an email from Takashi, it is sent from a workshop he is attending in some faraway place, with a note about what is coming up on his schedule. I wonder where his energy will take him (and the rest of us in probability in Japan) next! For now, though, I will finish with a simple message that I think many people would join in with: congratulations on the award!

References

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