日　本　数　学　会

２００２年度年会

函数論分科会
講演アブストラクト

11階

２００２年３月

於　明治大学駿河台校舎

函数論分科会委員会規則

1. 函数論分科会委員会（以下，委員会と略）の目的
函数論研究者の研究活動を活発にし，研究討論及び研究連絡を円滑に行うことを目的とする．

2. 委員会の任務
   (1) 函数論分科会評議員候補者を選出する．
   (2) 数学会より依頼された分科会選出の各種委員 (たとえば，受賞候補推薦委員等) 候補者の推薦．
   (3) 科研費基盤研究（審査区分（１））の代表者の推薦および計画調書提出の依頼．
   (4) 科研費運用に関して代表者または分担者から相談された時はこれに応ずる．
   (5) 学会特別講演およびシンポジウム講演の講演候補者を順位を付して決定する．
   (6) 分科会の行事 (たとえば， シンポジウムの開催等) について決定する．
   (7) 次期委員会委員候補者の推薦．
   (8) その他評議員の要請する案件および分科会に関する一切の問題を協議決定する．

3. 委員会の構成及び委員の選出·任期
   (1) 委員の定数は特に定めないが10名程度をもって構成する．必要に応じ追加， 削減できる．
   (2) 委員の任期は春季学会から２年間とする．再任は妨げないが，原則として再々任は認めない．
   (3) 委員の選出は秋季学会において投票によって行う．
      (i) 委員会推薦の新任候補者について信任投票を行い，その結果，投票総数の過半数を得た候補者に委員を委嘱する．その際，函数論の研究分野のバランス，更に地区的にも偏しないよう候補者の推薦を配慮する．
      (ii) 投票の結果，委員会推薦候補者以外に分科会会員の１０名以上から推薦された者があるときには得票数上位２名に委員を委嘱する．

4. 委員会の開催及び議決
   (1) 委員会は評議員が召集する．
   (2) 委員会は委員総数の過半数の出席で成立する．
   (3) 年３回 (春季，シンポジウム，秋季) 定期委員会を開催する．必要に応じ臨時委員会を開催する事ができる．
   (4) 案件の議決は投票によってはならない．決定できない時は懸案事項として次回に繰越す．緊急事項については評議員に処置を一任する．

5. 函数論分科会委員会における評議員の任務
   (1) 委員会の司会をする．
   (2) 数学会評議員会の決定事項を委員会に報告する．
   (3) 委員会で決定した事項 (シンポジウム，学会特別講演等) を施行する．
   (4) 委員会の了承を得て，決定事項を分科会会員に公表する．

付 則　この規則は，1974年10月12日より施行する．
付 則　この規則の改正は，1996年8月1日より施行する．

第1日　3月28日 (木)

第VII会場　函数論

9:30~12:00

1	布川　護	(群馬大教育)	* Some properties of analytic functions at extremal points for arguments	15
	尾和　重義	(近畿大理工)		
	斎藤　斉	(群馬高専)		
	N. E. Cho	(釜慶大理)		
	高橋　典宏	(群馬大教育)		
2	B. MacCluer	(Univ. of Virginia)	* Topological structure of the space of composition operators on H^∞	15
	大野　修一	(日本工大工)		
	R. Zhao	(Univ. of Cincinnati)		
3	G. G. Gundersen	(Univ. of New Orleans)	* Unique range sets for polynomials or rational functions	15
	藤解　和也	(金沢大工)		
4	戸田　暢茂	(名工大)	* On the deficiency of holomorphic curves with maximal deficiency sum, III	15
5	二村　俊英	(広島大理)	* Tangential limits and removable sets for weighted Sobolev spaces	15
	水田　義弘	(広島大総合科)		
6	宮本　育子	(千葉大理)	* シリンダーでのボイリング-ダールベルグ-ショグレン型の定理	15
	柳下　稔	(千葉大自然)		
7	前田　文之	(広島工大)	* Perturbation theory for nonlinear Dirichlet problems	15
	小野　太幹	(福山大)		

14:15~14:45

8	中井　三留	(名工大)	無限葉平面の調和次元	15
9	相川　弘明	(島根大総合理工)	* Hölder coutinuity of Dirichlet solution for a general domain	15

15:00~17:15　特別講演

水田　義弘	(広島大総合科)	* 距離空間上のソボレフ関数について	(15:00~16:00)
須川　敏幸	(京大理)	* 退化 Beltrami 方程式　—古典的アプローチ—	(16:15~17:15)

第2日　3月29日 (金)

第VII会場　函数論

9:30~12:00

13:00~13:30

13:45~14:45　特別講演

梅野　高司　(九州産大工) * トロイダル群 $-\bar{\partial}$ コホモロジーと準アーベル多様体−

1. Some properties of analytic functions at extremal points for arguments

MAMORU NUNOKAWA (Gunma University)
SHIGEYOSHI OWA (Kinki University)
HITOSHI SAITOH (Gunma College of Technology)
NAK EUN CHO (Pukyong National University)
NORIHIRO TAKAHASHI (Gunma University)

Let $\mathcal{N}$ be the class of all functions $p(z)$ that are analytic in the open unit disc $E = \{z \in \mathbb{C} : |z| < 1\}$ with $p(0) = 1$. Let $\mathcal{A}$ denote the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

which are analytic in E.

Theorem 1. *Let $p(z) \in \mathcal{N}$ and $p(z) \neq 0$ in E. If there exist two points $z_1 \in E$ and $z_2 \in E$ such that*

$$-\frac{\pi\beta}{2} = \arg p(z_1) < \arg p(z) < \arg p(z_2) = \frac{\pi\alpha}{2}$$

for some α $(\alpha > 0)$, some β $(\beta > 0)$ and for all z $(|z| < |z_1| = |z_2|)$, then we have

$$\frac{z_1 p'(z_1)}{p(z_1)} = -i\,\frac{\alpha+\beta}{2}\,m$$

and

$$\frac{z_2 p'(z_2)}{p(z_2)} = i\,\frac{\alpha+\beta}{2}\,m$$

where

$$m \geq \frac{1-|a|}{1+|a|},$$

and

$$a = i\,\tan\frac{\pi}{4}\left(\frac{\alpha-\beta}{\alpha+\beta}\right).$$

Theorem 2. *If $f(z) \in \mathcal{A}$ satisfies $f(z) \neq 0$ $(0 < |z| < 1)$ and*

$$-\frac{\pi\beta}{2} - \mathrm{Tan}^{-1}\left(\frac{\alpha+\beta}{2}\right)\left(\frac{1-|a|}{1+|a|}\right) < \arg f'(z)$$
$$< \frac{\pi\alpha}{2} + \mathrm{Tan}^{-1}\left(\frac{\alpha+\beta}{2}\right)\left(\frac{1-|a|}{1+|a|}\right) \qquad (z \in E)$$

with

$$a = i \tan\frac{\pi}{4}\left(\frac{\alpha-\beta}{\alpha+\beta}\right),$$

then

$$-\frac{\pi\beta}{2} < \arg\frac{f(z)}{z} < \frac{\pi\alpha}{2} \qquad (z \in E),$$

where $\alpha > 0$ and $\beta > 0$.

Corollary 1. *If $f(z) \in \mathcal{A}$ satisfies $f(z) \neq 0$ $(0 < |z| < 1)$ and*

$$|\arg f'(z)| < \frac{\pi\alpha}{2} + \mathrm{Tan}^{-1}\alpha \qquad (z \in E),$$

then

$$|\arg\frac{f(z)}{z}| < \frac{\pi\alpha}{2} \qquad (z \in E),$$

where $\alpha > 0$.

Theorem 3. *If $f(z) \in \mathcal{A}$ satisfies $f'(z) \neq 0$ $(z \in E)$ and*

$$-\frac{\pi\beta}{2} - \mathrm{Tan}^{-1}\left(\frac{\alpha+\beta}{2}\right)\left(\frac{1-|a|}{1+|a|}\right) < \arg(zf'(z))'$$
$$< \frac{\pi\alpha}{2} + \mathrm{Tan}^{-1}\left(\frac{\alpha+\beta}{2}\right)\left(\frac{1-|a|}{1+|a|}\right) \qquad (z \in E)$$

with

$$a = i \tan\frac{\pi}{4}\left(\frac{\alpha-\beta}{\alpha+\beta}\right),$$

then

$$-\frac{\pi\beta}{2} < \arg f'(z) < \frac{\pi\alpha}{2} \qquad (z \in E),$$

where $\alpha > 0$ and $\beta > 0$.

Corollary 2. *If $f(z) \in \mathcal{A}$ satisfies $f'(z) \neq 0$ $(z \in E)$ and*

$$|\arg(zf'(z))'| < \frac{\pi}{2}\alpha + \mathrm{Tan}^{-1}\alpha \quad (z \in E),$$

then

$$|\arg f'(z)| < \frac{\pi}{2}\alpha \quad (z \in E),$$

where $\alpha > 0$.

2. Topological Structure of the Space of Composition Operators on H^∞

Barbara MacCluer University of Virginia
大野 修一 日本工業大・工学部
Ruhan Zhao University of Cincinnati

Let $\mathcal{H}(D)$ be the space of all analytic functions on the unit disk D. Let $\mathcal{S}(D)$ denote the set of all analytic self maps of the unit disk D. Every analytic self-map $\varphi \in \mathcal{S}(D)$ induces through composition a linear *composition operator* C_φ from $\mathcal{H}(D)$ to itself. Thus C_φ is defined by $C_\varphi(f) = f \circ \varphi$ for $f \in \mathcal{H}(D)$. For a Banach space X of analytic functions on the unit disk D, let $\mathcal{C}(X)$ denote the space of composition operators on X with topology induced by the operator norm. A central problem in the investigation of composition operators is to relate function theoretic properties of φ to operator theoretic properties of the restriction of C_φ to various Banach spaces of analytic functions. Here we study the topological structure of $\mathcal{C}(H^\infty)$, where H^∞ is the space of bounded analytic functions on D.

For the Hardy space H^2, the analogous problem has been considered by several authors. In 1981, Earl Berkson has discovered that certain highly non-compact composition operators on H^2 are isolated (that is, are singleton components) in the space $\mathcal{C}(H^2)$. The result has been further generalized by Barbara D. MacCluer (1989) and Joel H. Shapiro and Carl Sundberg (1990). Then J. H. Shapiro and C. Sundberg raised the following problems:

1. *Characterize the components of* $\mathcal{C}(H^2)$.
2. *Which composition operators are isolated in* $\mathcal{C}(H^2)$*?*
3. *Which composition differences are compact on* H^2*?*

These problems seem quite hard. For the Problem 1, however, Shapiro and Sundberg suggest the following conjecture:

The set of all composition operators that differ from the given one by a compact operator forms a component in $\mathcal{C}(H^2)$.

We are not intending to solve the above problems for H^2. Instead, we will study these problems on the simpler setting H^∞. We hope that our investigation for the case of H^∞ may give some clues for solving the original problems in the setting of H^2. For the setting of H^∞, we are able to solve the above problems almost completely. A surprising result is that, a component in $\mathcal{C}(H^\infty)$ is *not* in general the set of all composition operators that differ from the given one by a compact operator. Thus, Shapiro and Sundberg's conjecture is not true for the setting of H^∞.

Our results involve the pseudo-hyperbolic metric. For $z, w \in D$, the pseudo-hyperbolic distance between z and w is given by

$$\beta(z,w) = \left|\frac{z-w}{1-\bar{z}w}\right|.$$

We will also use the hyperbolic metric which is given by the following formula:

$$\rho(z,w) = \frac{1}{2}\log\frac{1+\beta(z,w)}{1-\beta(z,w)}.$$

In the next result, and throughout the paper, $\|T\|$ denotes the norm of an operator T on H^∞.

Our main results are as follows.

Theorem 1. Let φ and ψ be analytic self maps of D. Then the following conditions are equivalent:

(i) C_φ and C_ψ are in the same path component in $\mathcal{C}(H^\infty)$.

(ii) $\|C_\varphi - C_\psi\| < 2$.

The next result deals with compact composition differences on H^∞. Let $\mathcal{B}$ be the Bloch space.

Theorem 2. Let φ and ψ be analytic self maps of the unit disk D, and let $\varphi \neq \psi$. Then the following conditions are equivalent:

(i) $C_\varphi - C_\psi : H^\infty \to H^\infty$ is compact;

(ii) $C_\varphi - C_\psi : B \to H^\infty$ is compact;

(iii) one of (a) or (b) holds:

(a) $\partial\varphi(D) \cap \partial D = \partial\psi(D) \cap \partial D = \emptyset$;

(b) $\partial\varphi(D) \cap \partial D = \partial\psi(D) \cap \partial D \neq \emptyset$ and

$$\lim_{|\varphi(z)|\to 1} \beta(\varphi(z), \psi(z)) = \lim_{|\psi(z)|\to 1} \beta(\varphi(z), \psi(z)) = 0.$$

3. Unique range sets for polynomials or rational functions

Gary G. Gundersen (University of New Orleans)
Kazuya Tohge (Kanazawa University)

We discuss what is known about *unique range sets* for polynomials or rational functions. We also discuss related results, including (i) rational functions that share three values, and (ii) sets which are almost (apart from exceptional cases) unique range sets for different classes of meromorphic functions.

Let $\mathcal{F}$ be a family of non-constant meromorphic functions defined on the plane **C**. Let S be a discrete subset of **C**. For each $f \in \mathcal{F}$, we define $E_f(S) = \bigcup_{a \in S}\{z : f(z) - a = 0\}$, where a zero of $f(z) - a$ of multiplicity m appears m times. The set S is called a **URSF**, *unique range set* for the family $\mathcal{F}$, provided that the following condition holds: If $f, g \in \mathcal{F}$ satisfy $E_f(S) = E_g(S)$, then $f \equiv g$. The condition $E_f(S) = E_g(S)$ expresses that f and g *share* the set S *CM* (counting multiplicities).

We denote by $\mathcal{E}$, $\mathcal{M}$, $\mathcal{P}$ and $\mathcal{R}$ the families of all non-constant entire functions, meromorphic functions on **C**, polynomials and rational functions, respectively, and we denote a unique range set for $\mathcal{E}$, $\mathcal{M}$, $\mathcal{P}$ or $\mathcal{R}$ by a **URSE**, **URSM**, **URSP** or **URSR**, respectively. Examples of *finite* URSEs and URSMs have been obtained by Yi, Li and Yang, Mues and Reinders, Fujimoto, Frank and Reinders, Shirosaki, and Shiffman. The existence of these finite sets leads naturally to the problem of trying to minimize the number of their elements. Let

$$\lambda(\mathcal{F}) := \min\Big\{|S| \;:\; S \text{ is a unique range set for the family } \mathcal{F}\Big\},$$

where $|S|$ denotes the cardinality of S. Li and Yang, and Yi gave examples which show that $\lambda(\mathcal{M}) \geq 5$ and $\lambda(\mathcal{E}) \geq 4$, and then Hua and Yang later gave examples which show that $\lambda(\mathcal{M}) \geq 6$ and $\lambda(\mathcal{E}) \geq 5$. Therefore, we see that $6 \leq \lambda(\mathcal{M}) \leq 11$ and $5 \leq \lambda(\mathcal{E}) \leq 7$.

We will show that $\lambda(P) = 3$ and $5 \leq \lambda(\mathcal{R}) \leq 10$. Hu and Yang gave an example of a URSR with *ten* elements. Hu and Yang also proved the following result, which shows that $\lambda(\mathcal{P}) \neq 2$, that is, there cannot exist a URSP with two distinct elements.

Theorem 1 *If $f \in \mathcal{M}$, and if $g := -f - a$, then $f^2 + af + b = g^2 + ag + b$. Thus there cannot exist a URSP with two elements.*

Conversely, suppose that P, $Q \in \mathcal{P}$ share the set $\{z : z^2 + az + b = 0\}$ $(a^2 \neq 4b)$ CM, where $P \not\equiv Q$. Then $P \equiv -Q - a$.

Next we prove that $\lambda(\mathcal{P}) = 3$, that is, there is a three point URSP.

Theorem 2 *For any $a,\ b \in \mathbf{C} \setminus \{0\}$, the set $S := \{w : w^3 + aw + b = 0\}$ is a URSP, whenever S has three distinct elements.*

Remark 1 Theorem 2 is equivalent to a characterization of Boutabaa, Escassut and Haddad in terms of affinely rigid sets (or stiff sets).

Now we see that $\lambda(\mathcal{R}) \geq 3$, as a corollary of Theorem 1. Pakovitch gave an example which shows that $\lambda(\mathcal{R}) \geq 4$. We will show that $\lambda(\mathcal{R}) \geq 5$.

If $f,\ g \in \mathcal{R}$ share a set $S = \{\alpha\}$ that consists of one element α, then we say that f and g share the value α. A shared value can be by *CM* (counting multiplicities), by *IM* (ignoring multiplicities), or by *DM* (by different multiplicities at every α-point of f and g).

Gross proved that if $f,\ g \in \mathcal{R}$ share three values CM, then $f \equiv g$. We prove the following improvement of this theorem:

Theorem 3 *If $f,\ g \in \mathcal{R}$ share two values CM and one value IM, then $f \equiv g$.*

Examples show that we cannot replace "two values CM and one value IM" with "one value CM and two values IM" in the hypothesis of Theorem 3 . We also give an example of two rational functions that share three values DM.

Adams and Straus proved that if $f,\ g \in \mathcal{R}$ share four values IM, then $f \equiv g$.

We will now give an example of a 10-point set Φ which is *not* a URSR, but except for one exceptional pair (f, g), we will see that $f \equiv g$ whenever f, $g \in \mathcal{R}$ share Φ CM. We give several consequences of this result and related results. One of the consequences proves that $\lambda(\mathcal{R}) \geq 5$.

Theorem 4 *Let $a, b \in \mathbf{C} \setminus \{0\}$ be chosen so that the set $\Phi = \{z : z^{10} + az^9 + b = 0\}$ has ten distinct elements. If $f,\ g \in \mathcal{R}$ share Φ CM, then we have $f \equiv g$ unless*

$$f(z) = \frac{ap(z)\{p^9(z) - \omega q^9(z)\}}{q^{10}(z) - p^{10}(z)}, \quad g(z) = \frac{aq(z)\{p^9(z) - \omega q^9(z)\}}{\omega\{q^{10}(z) - p^{10}(z)\}}, \tag{1}$$

where p and q are polynomials and ω is a 10th root of unity.

To prove this result, we apply a particular case of the Cartan-Hayman theorem.

Remark 2 1) Yi obtained a result for the family $\mathcal{M}$ that is similar in nature to Theorem 4 , and Yi used his result to give examples of finite sets which are URSEs but not URSMs.

2) The two rational functions in (1) share ∞ CM. We use this observation to show that any set of *four* elements cannot be a URSR.

4. On the deficiency of holomorphic curves with maximal deficiency sum, III

TODA Nobushige   Nagoya Institute of Technology

1. Let $f = [f_1, \cdots, f_{n+1}]$ be a trascendental and linearly non-degenerate holomorphic curve from $\boldsymbol{C}$ into $P^n(\boldsymbol{C})$ with a reduced representation

$$(f_1, \cdots, f_{n+1}) : \boldsymbol{C} \to \boldsymbol{C}^{n+1} - \{\mathbf{0}\},$$

where n is a positive integer.

Let X be a subset of $\boldsymbol{C}^{n+1} - \{\mathbf{0}\}$ in N-subgeneral position; that is to say, (i) $\#X \geq N+1$ and (ii) any $N+1$ elements of X generate $\boldsymbol{C}^{n+1}$, where N is an integer satisfying $N \geq n$.

We denote by $T(r, f)$ the characteristic function of f and by $\delta(\boldsymbol{a}, f)$ the deficiency of $\boldsymbol{a} \in \boldsymbol{C}^{n+1} - \{\mathbf{0}\}$ with respect to f.

H. Cartan([1],$N = n$) and E. I. Nochka([2],$N > n$) proved the following

Defect relation. $\sum_{\boldsymbol{a} \in X} \delta(\boldsymbol{a}, f) \leq 2N - n + 1$.

We are interested in the holomorphic curves extremal in the defect relation.

Theorem A([3], [4]). Suppose that

(i) $N > n = 2m$ $(m \in \boldsymbol{N})$ and (ii) $\sum_{\boldsymbol{a} \in X} \delta(\boldsymbol{a}, f) = 2N - n + 1$.

Then, there are at least $[(2N - n + 1)/(n+1)] + 1$ vectors $\boldsymbol{a} \in X$ satisfying $\delta(\boldsymbol{a}, f) = 1$.

The purpose of this talk is to give a theorem when $n = 2m - 1$.

2. Let q be an integer satisfying $2N - n + 1 < q < \infty$ and we put $Q = \{1, 2, \cdots, q\}$. Let $\{\boldsymbol{a}_j \mid j \in Q\}$ be a subset of X. For a non-empty subset P of Q, we denote

$$V(P) = \text{the vector space spanned by } \{\boldsymbol{a}_j \mid j \in P\}, \quad d(P) = \dim V(P),$$

$$\mathcal{O} = \{P \subset Q \mid 0 < \#P \leq N+1\}.$$

Further we put $\lambda = \min_{P \in \mathcal{O}} d(P)/\#P$. Then

$1/(N - n + 1) \leq \lambda \leq (n+1)/(N+1)$ and

$$\sum_{j=1}^{q} \delta(\boldsymbol{a}_j, f) \leq \min(2N - n + 1, (n+1)/\lambda). \tag{1}$$

Theorem. Suppose that

(i) $N > n = 2m - 1$ and $(N+1, m) = 1$ $(m \in N)$ and

(ii) there exist $\boldsymbol{a}_1, \cdots, \boldsymbol{a}_q \in X$ $(2N - n + 1 < q < \infty)$ satisfying $\delta(\boldsymbol{a}_j, f) > 0$ $(j = 1, \cdots, q)$ and

$$\sum_{j=1}^{q} \delta(\boldsymbol{a}_j, f) = 2N - n + 1.$$

Then, either (I) or (II) given below holds:

(I) There exist at least $[(2N-n+1)/(n+1)]+1$ integers $j \in Q$ satisfying $\delta(\boldsymbol{a}_j, f) = 1$.

(II) q is divisible by $N - m + 1$ and $Q = \cup_{\nu=1}^{p} P_\nu$,

where (a) $p = q/(N - m + 1)$, (b) $P_1, \cdots, P_p$ are mutually disjoint and (c) $d(P_\nu) = m, \#P_\nu = N - m + 1$ $(\nu = 1, \cdots, p)$.

Proof. From (1) and (ii) we obtain the inequality $\lambda \leq (n+1)/(2N - n + 1)$, which is equal to $m/(N - m + 1)$ by (i).

When $\lambda < m/(N - m + 1)$, we have (I) and when $\lambda = m/(N - m + 1)$, we have (II).

Remark. We can prove a similar result to this theorem when $q = \infty$.

References

[1] H. Cartan : Sur les combinaisons linéaires de p fonctions holomorphes données. Mathematica **7**(1933), 5-31.

[2] E. I. Nochka : On the theory of holomorphic functions. Soviet Math. Dokl., **27**-2(1983), 377-381.

[3] N. Toda : On the deficiency of holomorphic curves with maximal deficiency sum. Kodai Math. J., **24**-1(2001), 134-146.

[4] N. Toda : On the deficiency of holomorphic curves with maximal deficiency sum, II. Proceedings of the 3rd ISAAC Congress (to appear).

5. Tangential limits and removable sets for weighted Sobolev spaces

二村俊英　広島大大学院・理学研究科
水田義弘　広島大学・総合科学部

$p > 1$, $-1 < \alpha < p-1$ に対して,

$$d\mu_\alpha(x) = |x_n|^\alpha dx \quad (x = (x', x_n) \in \mathbf{R}^n = \mathbf{R}^{n-1} \times \mathbf{R})$$

を重みとするソボレフ空間 $W^{1,p}(\Omega; \mu_\alpha)$ を考える.

Ω を $\mathbf{R}^n$ 内の開集合で, E を測度零の Ω 内の部分閉集合とする. このとき, 集合 E が $W^{1,p}(\Omega; \mu_\alpha)$ で除去可能であるとは, $W^{1,p}(\Omega \setminus E; \mu_\alpha) = W^{1,p}(\Omega; \mu_\alpha)$ となるときを言う. すなわち, 任意の $u \in W^{1,p}(\Omega \setminus E; \mu_\alpha)$ に対して, $\Omega \setminus E$ 上ほとんど至る所で $u = u_0$ となる $u_0 \in W^{1,p}(\Omega; \mu_\alpha)$ が存在する. 一般に, 除去可能集合は quasiconvex であることが知られている. 本講演の目的は, 超平面 $\mathbf{R}^{n-1} = \mathbf{R}^{n-1} \times \{0\}$ 内にある集合 E が $W^{1,p}(\mathbf{R}^n; \mu_\alpha)$ で除去可能であるための十分条件を与えることである.

命題. $1 < p \leq n + \alpha$, $-1 < \alpha < p - 1$ で, 集合 E を $\mathbf{R}^{n-1}$ 内の部分閉集合とする. このとき, 次は同値である.

(1) E は $W^{1,p}(\mathbf{R}^n; \mu_\alpha)$ で除去可能である;

(2) 任意の $\mathbf{R}^n \setminus E$ 上の (p, α)-仮似連続関数 $u \in W^{1,p}(\mathbf{R}^n \setminus E; \mu_\alpha)$ に対して, $\mathcal{H}^{n-1}$- a.e. $(x', 0) \in E$ で次が成り立つ:

$$\lim_{0<t\to 0} u(x', t) = \lim_{0>t\to 0} u(x', t);$$

(3) 任意の $\mathbf{R}^n \setminus E$ 上の (p, α)-仮似連続関数 $u \in W^{1,p}(\mathbf{R}^n \setminus E; \mu_\alpha)$ に対して, $\mathcal{H}^{n-1}$- a.e. $\xi \in E$ で次が成り立つ:

$$\lim_{\Gamma(\xi,a)\ni x\to\xi} \fint_{B(x,x_n/2)} u\, d\mu_\alpha = \lim_{\Gamma(\xi,a)\ni \hat{x}\to\xi} \fint_{B(x,-x_n/2)} u\, d\mu_\alpha \quad (\forall a > 0).$$

ここに, $x = (x', x_n)$ に対して $\hat{x} = (x', -x_n)$ とし, $\xi \in \mathbf{R}^{n-1}$, $a > 0$ に対して,

$$\Gamma(\xi, a) = \begin{cases} \{x \in \mathbf{R}^n_+ : |x - \xi|^{\frac{n-1}{n+\alpha-p}} < a x_n\} & (1 < p < n + \alpha) \\ \{x \in \mathbf{R}^n_+ : |x - \xi| \exp(-|x - \xi|^{-\frac{n-1}{n+\alpha-1}}) < a x_n\} & (p = n + \alpha) \end{cases}$$

とする.

定義. $E \subset \mathbf{R}^{n-1}$ が (p, α)-porous であるとは, $\mathcal{H}^{n-1}$- a.e. $x \in E$ に対して, 次のような $r_i = r_i(x)$, $r_i \to 0$, と定数 $C_x > 0$ が存在するときを言う:

(i) $1 < p \leq n+\alpha-1$ のとき，各 i に対して $B_i \subset B^{n-1}(x,r_i) \setminus E$ で $R_i = \mathrm{diam}(B_i) \geq C_x r_i^{(n-1)/(n+\alpha-p)}$ となる球 B_i が存在する．

(ii) $n+\alpha-1 < p < n+\alpha$ のとき，各 i に対して $F_i \subset B^{n-1}(x,r_i) \setminus E$ で $R_i = \mathrm{diam}(F_i) \geq C_x r_i^{(n-1)/(n+\alpha-p)}$ となる連続体 F_i が存在する．

(iii) $p = n+\alpha$ のとき，各 i に対して $F_i \subset B^{n-1}(x,r_i) \setminus E$ で $R_i = \mathrm{diam}(F_i) \geq C_x r_i \exp(-C_x r_i^{(n-1)/(1-n-\alpha)})$ となる連続体 F_i が存在する．

ここに，$B^{n-1}(x,r) = B(x,r) \cap \mathbf{R}^{n-1}$, $B(x,r) = \{y \in \mathbf{R}^n : |x-y| < r\}$ とする．

主定理. $1 < p \leq n+\alpha$, $-1 < \alpha < p-1$ で，集合 E を $\mathbf{R}^{n-1}$ 内の部分閉集合とする．このとき，E が (p,α)-porous であるならば，E は $W^{1,p}(\mathbf{R}^n; \mu_\alpha)$ で除去可能である．

注意. $p > n+\alpha$ のとき，$\mathbf{R}^n$ 上の (p,α)-仮似連続関数 $u \in W^{1,p}(\mathbf{R}^n; \mu_\alpha)$ の超平面 $\mathbf{R}^{n-1}$ への制限 $u|\mathbf{R}^{n-1}$ は，$(p-n-\alpha)/p$ - 次の Hölder 連続な関数となるので次のことが言える：$E \subset \mathbf{R}^{n-1}$ が除去可能であるための必要十分条件は，E が内点を持たないことである．

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6. シリンダーでのボイリング - ダールベルグ - ショグレン型の定理

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D を $\mathbf{R}^{n-1}(n \geq 2)$ 上の滑らかな境界をもつ有界領域とし,

$$\Gamma_n(D) = \{(X, y) \in \mathbf{R}^n; X \in D, -\infty < y < +\infty\}$$

をシリンダーと呼ぶ.

ディリクレ問題

$$\begin{aligned} (\Delta_n + \tau)f &= 0 \quad \text{on } D \\ f &= 0 \quad \text{on } \partial D, \end{aligned}$$

の最小な正の固有値と, 正規化した固有関数をそれぞれ τ_D, $f_D(X)$ で表す.

$\Gamma_n(D)$ のマルチン境界は集合 $\partial\Gamma_n(D) \cup \{+\infty, -\infty\}$ であり, ある適当な基準点に関するマルチン核を $K(P, Q)$ $(P \in \Gamma_n(D), Q \in \partial\Gamma_n(D) \cup \{+\infty, -\infty\})$ で表す時,

$$K(P, +\infty) = e^{\sqrt{\tau_D}y} f_D(X), \quad (P = (X, y) \in \Gamma_n(D))$$

である事が知られている.

$\Gamma_n(D)$ の部分集合 E が $+\infty$ で minimally thin であるとは,

$$\hat{R}^E_{K(\cdot,+\infty)}(P) \neq K(P, +\infty),$$

なる $P \in \Gamma_n(D)$ が存在するときをいう.

$$\Gamma_n(D; 0, +\infty) = \{P = (X, y) \in \mathbf{R}^n; X \in D, y > 0\}$$

とする.

シリンダー内のボイリング - ダールベルグ - ショグレン型の次の定理を報告する.

定理 1. $\Gamma_n(D)$ のボレル部分集合 $E(\subset \Gamma_n(D; 0, +\infty))$ が $+\infty$ で minimally thin であるとき, E のルベーク測度を $|E|$ とすれば

$$|E| < \infty \tag{$*$}$$

が成り立つ. 更に, $E(\subset \Gamma_n(D; 0, +\infty))$ が $\Gamma_n(D)$ の Whitney cube からの cube の和であるとき, $(*)$ は E が $+\infty$ で minimally thin であることの十分な条件である.

この定理の証明には, 滑らかな境界を持つ有界領域における同種の結果の証明 ([1] 参照) を参考にし, [3] を用いる.

また E が $+\infty$ で minimally thin であることの同値な条件 ([3] 参照), 及び定理 1 を用いると以下の定理 2, 定理 3 を得る.

定理 2 (ダールベルグ [2] 型の定理). ボレル可測集合 $E(\subset \Gamma_n(D))$ が

$$|E| = \infty$$

を満たすと仮定する. $v(P)$ を $\Gamma_n(D)$ 上正値優調和な関数, m を正の数で E 上で $v(P) \geq mK(P, +\infty)$ を満たすとする. このとき $\Gamma_n(D)$ 全体で不等式 $v(P) \geq mK(P, +\infty)$ を満たす.

定理 3 (ショグレン [4] 型の定理). $v(P)$ を $\Gamma_n(D)$ 上の正値優調和関数で

$$\inf_{P \in \Gamma_n(D)} \frac{v(P)}{K(P, +\infty)} = 0$$

を満たすとする. このとき

$$|M_v| < \infty,$$

ただし,

$$M_v = \{P \in \Gamma_n(D); v(P) \geq K(P, +\infty\}.$$

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7. Perturbation theory for nonlinear Dirichlet problems

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Let Ω be a fixed domain in $\mathbf{R}^N$ $(N \geq 2)$ and as in [MO1] and [MO2] we consider a quasi-linear elliptic differential equation

$$(\mathrm{E}_{\mathcal{A},\mathcal{B}}) \qquad -\operatorname{div} \mathcal{A}(x, \nabla u(x)) + \mathcal{B}(x, u(x)) = 0$$

on Ω. Here, $\mathcal{A} : \Omega \times \mathbf{R}^N \to \mathbf{R}^N$ and $\mathcal{B} : \Omega \times \mathbf{R} \to \mathbf{R}$ satisfy the following conditions for $1 < p < \infty$ and a *weight* w which is *p-admissible* in the sense of [HKM]:

(A.1) $x \mapsto \mathcal{A}(x, \xi)$ is measurable on Ω for every $\xi \in \mathbf{R}^N$ and $\xi \mapsto \mathcal{A}(x, \xi)$ is continuous for a.e. $x \in \Omega$;

(A.2) $\mathcal{A}(x, \xi) \cdot \xi \geq \alpha_1 w(x) |\xi|^p$ for all $\xi \in \mathbf{R}^N$ and a.e. $x \in \Omega$ with a constant $\alpha_1 > 0$;

(A.3) $|\mathcal{A}(x, \xi)| \leq \alpha_2 w(x) |\xi|^{p-1}$ for all $\xi \in \mathbf{R}^N$ and a.e. $x \in \Omega$ with a constant $\alpha_2 > 0$;

(A.4) $\big(\mathcal{A}(x, \xi_1) - \mathcal{A}(x, \xi_2)\big) \cdot \big(\xi_1 - \xi_2\big) > 0$ whenever $\xi_1, \ \xi_2 \in \mathbf{R}^N$, $\xi_1 \neq \xi_2$, for a.e. $x \in \Omega$;

(B.1) $x \mapsto \mathcal{B}(x, t)$ is measurable on Ω for every $t \in \mathbf{R}$ and $t \mapsto \mathcal{B}(x, t)$ is continuous for a.e. $x \in \Omega$;

(B.2) For any open set $D \Subset \Omega$, there is a constant $\alpha_3(D) \geq 0$ such that $|\mathcal{B}(x, t)| \leq \alpha_3(D) w(x)(|t|^{p-1} + 1)$ for all $t \in \mathbf{R}$ and a.e. $x \in D$;

(B.3) $t \mapsto \mathcal{B}(x, t)$ is nondecreasing on $\mathbf{R}$ for a.e. $x \in \Omega$.

A continuous solution of $(\mathrm{E}_{\mathcal{A},\mathcal{B}})$ in an open set $D \subset \Omega$ is called $(\mathcal{A}, \mathcal{B})$-*harmonic* in D. We consider the following function spaces:

$$\mathcal{D}^p(\Omega; \mu) = \{f \in H^{1,p}_{\mathrm{loc}}(\Omega; \mu) \,|\, |\nabla f| \in L^p(\Omega; \mu), f \text{ is bounded continuous}\},$$

$$\mathcal{D}^p_0(\Omega; \mu) = \left\{ f \in \mathcal{D}^p(\Omega; \mu) \;\middle|\; \begin{array}{l} \text{there exist } \varphi_n \in C_0^\infty(\Omega) \text{ such that } \varphi_n \to f \text{ a.e. ,} \\ \{\varphi_n\} \text{ is uniformly bounded, } \nabla \varphi_n \to \nabla f \text{ in } L^p(\Omega; \mu) \end{array} \right\},$$

where μ is a measure defined by $d\mu(x) = w(x)dx$. We say that Ω is (p, μ)-hyperbolic if $1 \notin \mathcal{D}^p_0(\Omega; \mu)$. In this paper, we always assume that Ω is (p, μ)-hyperbolic

We consider the following function spaces:

$$\mathcal{F}_b(\mathcal{A}) = \left\{ f \in L^1(\Omega) \;\middle|\; \begin{array}{l} f/w \text{ is locally bounded in } \Omega \text{ and} \\ -\operatorname{div} \mathcal{A}(x, \nabla u) = f \text{ has a solution in } \mathcal{D}^p_0(\Omega; \mu) \end{array} \right\},$$

$\mathcal{F}_b^+(\mathcal{A}) = \{f \in \mathcal{F}_b(\mathcal{A}) \,|\, f \geq 0\}$ and $\mathcal{F}_b^-(\mathcal{A}) = \{f \in \mathcal{F}_b(\mathcal{A}) \,|\, f \leq 0\}$.

For $f \in \mathcal{F}_b(\mathcal{A})$, the solution of $-\mathrm{div}\,\mathcal{A}(x, \nabla u) = f$ in $\mathcal{D}_0^p(\Omega; \mu)$ will be denoted by U^f.

In addition to (B.1), (B.2) and (B.3), we shall always assume that $\mathcal{B}$ satisfies the following condition (B.4) and (B.5):

(B.4) There exist nonnegative numbers T_1, T_2, functions $f_1 \in \mathcal{F}_b^+(\mathcal{A})$ and $f_2 \in \mathcal{F}_b^-(\mathcal{A})$ such that $\mathcal{B}^-(x, T_1) \leq f_1(x)$ and $\mathcal{B}^+(x, -T_2) \leq -f_2(x)$ a.e. in Ω.

(B.5) $\displaystyle\int_\Omega |\mathcal{B}(x,t)|dx < \infty$ for any $t \in \mathbf{R}$.

Theorem 1. *Let $\theta \in \mathcal{D}^p(\Omega; \mu)$. Then , there exists a unique $(\mathcal{A}, \mathcal{B})$-harmonic function $u_{(\mathcal{A},\mathcal{B},\theta)}$ on Ω such that $u_{(\mathcal{A},\mathcal{B},\theta)} - \theta \in \mathcal{D}_0^p(\Omega; \mu)$. Further it satisfies*

$$\min(-T_2, \inf_{\partial\Omega} \theta) + U^{f_2}(x) \leq u_{(\mathcal{A},\mathcal{B},\theta)}(x) \leq \max(T_1, \sup_{\partial\Omega} \theta) + U^{f_1}(x)$$

on Ω. Here, $\sup_{\partial\Omega} \theta = \inf\{k \,|\, (\theta - k)^+ \in \mathcal{D}_0^p(\Omega; \mu)\}$ *and* $\inf_{\partial\Omega} \theta = \sup\{k \,|\, (\theta - k)^- \in \mathcal{D}_0^p(\Omega; \mu)\}$.

Theorem 2. *Suppose $\mathcal{B}_n$, $n = 1, 2, \ldots$ and $\mathcal{B}$ all satisfy* (B.4) *with the same T_1, T_2, $f_1 \in \mathcal{F}_b^+(\mathcal{A})$, $f_2 \in \mathcal{F}_b^-(\mathcal{A})$. Let $\theta \in \mathcal{D}^p(\Omega; \mu)$. Assume further that there exists a nonnegative measurable function $b(x)$ on Ω such that $b(x)/w(x)$ is locally bounded in Ω and*

$$\mathcal{B}_n^+(x, M_1) + \mathcal{B}_n^-(x, -M_2) \leq b(x) \quad \text{a.e. on } \Omega$$

for all n, where $M_1 = \max(T_1, \sup_{\partial\Omega} \theta) + \sup_\Omega U^{f_1}$ *and* $M_2 = \max(T_2, -\inf_{\partial\Omega} \theta) - \inf_\Omega U^{f_2}$. *If*

$$\int_\Omega \sup_{-M_2 \leq t \leq M_1} |\mathcal{B}_n(x,t) - \mathcal{B}(x,t)|\, dx \to 0 \quad (n \to \infty) \quad ,$$

then $u_{(\mathcal{A},\mathcal{B}_n,\theta)} \to u_{(\mathcal{A},\mathcal{B},\theta)}$ as $n \to \infty$ locally uniformly on Ω.

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8. 無限葉平面の調和次元

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開リーマン面Rのマルチン極小境界点の全体$\Delta_1(R)$の濃度card $\Delta_1(R)$をRの**調和次元**と呼び (Heins [4]) 記号dim Rで表す (即ち，dim $R :=$ card $\Delta_1(R)$) と，常に$1 \leq$ dim $R \leq \aleph$ (連続体濃度) であるので，dimは開リーマン面の族から濃度の閉区間$[1, \aleph]$ (即ち，$1 \leq \xi \leq \aleph$となる濃度ξの全体) への写像と考えられる．だから開リーマン面のある族$\mathcal{F}$の写像dimの値域をdim $\mathcal{F}$とかく (即ち，dim $\mathcal{F} := \{\text{dim } R : R \in \mathcal{F}\}$)．理想境界成分が唯一つである放物面 (即ち，族$O_G$に入る面) Rを**Heins 面**と呼びその全体を$\mathcal{H}$と記すとき，dim $\mathcal{H}$を決定せよと言うのが**Heins の問題**であり現時点での最良の結果は

$$\text{dim } \mathcal{H} \supset \mathrm{N} \cup \{\aleph_0, \ \aleph\} \tag{1}$$

である (発表順に[4],[5],[3],[2],[9],[7]等の結果の綜合；統一的証明については[8]参照)，但し，Nは正整数の全体で，$\aleph_0$は可算無限濃度とする．

複素平面Cの正規被覆面R，詳しくは三揃え(R, C, π)，の集合を$\mathcal{C}$とかく．ここで正規性は射影πが次の性質を持つ事とする (Ahlfors-Sario [1])：各$a \in \mathrm{C}$に対し，中心a半径$0 < r < \infty$の適当な円板$V := \Delta(a, r)$で，$\pi^{-1}(V)$の各成分が相対完閉となるものがとれる．この時各$R \in \mathcal{C}$の葉数$\sigma(R)$が定まり，葉数$\xi \in \mathrm{N} \cup \{\aleph_0\}$のCの正規被覆面の全体を$\mathcal{C}_\xi := \{R \in \mathcal{C} : \sigma(R) = \xi\}$と記すと．$\mathcal{C} = (\cup_{p \in \mathrm{N}} \mathcal{C}_p) \cup \mathcal{C}_{\aleph_0}$．今回は**多葉平面の Heins 問題**を論ずる：$\dim(\mathcal{H} \cap \mathcal{C})$を決定せよ．これに関しては，$p \in \mathrm{N}$のとき，Heins [4] 自身による結果

$$\dim(\mathcal{H} \cap \mathcal{C}_p) \subset [1, p] \tag{2}$$

及びMasaoka-Segawa [6]に代表される上の結果(2)の精密化

$$\dim(\mathcal{H} \cap \mathcal{C}_p) = [1, p] \tag{3}$$

に係わる一連の詳しい研究がある．しかし調和次元については，有限葉平面族$\mathcal{C}_p$ ($p \in \mathrm{N}$) の場合に較べて無限葉平面族$\mathcal{C}_{\aleph_0}$の場合は大変難しくあまり成果の集積は多くないように思われるが，ここでは上の(2)と(3)，特に，(3)に注目した素朴な疑問として，その形式的な類似（類推）（即ち，(3)の$p \in \mathrm{N}$を単に$\aleph_0$に置き換えた）

$$\text{(?)} \qquad \dim(\mathcal{H} \cap \mathcal{C}_{\aleph_0}) = [1, \aleph_0]$$

は正しいか否かを問題にする．答えは否であり，次の結果を得たので報告する：

主定理：　族$\mathcal{H} \cap \mathcal{C}_{\aleph_0}$は葉数$\aleph_0$を越えた調和次元$\aleph$を持つCの正規被覆Heins面を含み，更に次の(1)の精密化が成り立つ：

$$\text{(4)} \qquad \dim(\mathcal{H} \cap \mathcal{C}_{\aleph_0}) \supset \mathrm{N} \cup \{\aleph_0, \aleph\}.$$

無論(1)同様(4)でも，包含関係は連続体仮説の成立を仮定すると等号で置き換えられ，限定的な場合も含めてHeinsの問題は完全な解決を見るが，仮定しない時は全然未解決であり，(1)より(4)の方が具体性が高い分御し易いのではないかとの期待が，$\mathcal{H}$の代わりに$\mathcal{H} \cap \mathcal{C}$を考える動機の一つである．

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9. Hölder continuity of Dirichlet solution for a general domain

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Let $0 < \alpha \le 1$. For an arbitrary set $E \subset \mathbb{R}^n$, $n \ge 2$, we consider the family $\Lambda_\alpha(E)$ of all bounded α-Hölder continuous functions on E. The norm of $f \in \Lambda_\alpha(E)$ is given by

$$\|f\|_{\Lambda_\alpha(E)} = \sup_{x \in E} |f(x)| + \sup_{\substack{x,y \in E \\ x \neq y \\ |x-y| \le 1}} \frac{|f(x) - f(y)|}{|x-y|^\alpha}.$$

Let D be a bounded domain. By $\mathscr{H}(D)$ we denote the family of all harmonic functions on D. For a function f on ∂D we denote by $H_D f$ the Dirichlet (PWB-) solution of f over D, provided the solution exists. It is well known that if D is regular, then H_D maps $C(\partial D)$ to $\mathscr{H}(D) \cap C(\overline{D})$. It may be natural to think that the better continuity of a boundary function f ensures the better continuity of $H_D f$. In this paper we investigate conditions for H_D to map $\Lambda_\alpha(\partial D)$ to $\mathscr{H}(D) \cap \Lambda_\alpha(D)$ with the same Hölder exponent α. In other words, we study the conditions for $\|H_D\|_\alpha < \infty$, where

$$\|H_D\|_\alpha = \sup_{\substack{f \in \Lambda_\alpha(\partial D) \\ \|f\|_{\Lambda_\alpha(\partial D)} \neq 0}} \frac{\|H_D f\|_{\Lambda_\alpha(D)}}{\|f\|_{\Lambda_\alpha(\partial D)}}.$$

Hinkkanen [1] and Sugawa [2] considered similar problems mainly for planar domains in somewhat different context.

We observe that the Lipschitz continuity cannot be preserved by H_D. Sugawa [2, Example 6.1] observed this for a unit disc $D = B(0,1) \subset \mathbb{R}^2$: If $f(\xi) = f(\xi_1, \xi_2)$ is the Lipschitz continuous function $|\xi_2|$ on ∂D, then

$$H_D f(x_1, 0) = \frac{1 - x_1^2}{\pi x_1} \log \frac{1 + x_1}{1 - x_1}$$

and the Lipschitz continuity of $H_D f$ breaks down near $(\pm 1, 0)$. A similar example applies to the higher dimensional case. More strongly, however, we can show the following assertion.

Theorem 1. *There is no bounded domain for which* $\|H_D\|_1 < \infty$.

We write $\omega(\cdot, E, U)$ for the harmonic measure over an open set U of $E \subset \partial U$. Define a boundary function $\varphi_{a,\alpha}$ for $a \in \partial D$ by $\varphi_{a,\alpha}(\xi) = \min\{|\xi - a|^\alpha, 1\}$ for $\xi \in \partial D$. Then we have the following.

Theorem 2. *Let $0 < \alpha < 1$ and let D be a bounded regular domain. Consider the following conditions:*

(i) $\|H_D\|_\alpha < \infty$.

(ii) *For every $a \in \partial D$, $H_D\varphi_{a,\alpha}(x) \le M|x-a|^\alpha$.*

(iii) *For every $a \in \partial D$ and $r > 0$*

$$\omega(x, D \cap S(a,r), D \cap B(a,r)) \le M\left(\frac{|x-a|}{r}\right)^\alpha \quad \text{for } x \in D \cap B(a,r).$$

(iv) *For every $a \in \partial D$ and $r > 0$*

$$\omega(x, \partial D \setminus B(a,r), D) \le M\left(\frac{|x-a|}{r}\right)^\alpha \quad \text{for } x \in D.$$

Then we have

$$\text{(i)} \iff \text{(ii)} \implies \text{(iii)} \iff \text{(iv)}.$$

If (iii) *holds with some $\alpha' > \alpha$, then* (i) *holds.*

Corollary 1. *Let D be a bounded regular domain. Then $\|H_D\|_\alpha < \infty$ for some $\alpha > 0$ if and only if ∂D satisfies the capacity density condition, which is equivalent to the* uniform perfectness *of ∂D if $n = 2$.*

参考文献

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特別講演

距離空間上のソボレフ関数について

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目次

1 ポアンカレの不等式

$\mathbf{R}^n$ の球体 B 上の**ソボレフ関数** $u \in W^{1,p}(B)$ に対して，不等式

$$\fint_B |u - u_B|^p dx \le C(n,p)(\mathrm{diam}\ B)^p \fint_B |\nabla u|^p dx \tag{1.1}$$

は，通常，**ポアンカレの不等式**と呼ばれる。ここに，$1 \le p < \infty$ で

$$u_B = \fint_B u dx = \frac{1}{|B|}\int_B u dx$$

は積分平均を表す。また，不等式

$$\left(\fint_B |u - u_B|^{np/(n-p)} dx\right)^{(n-p)/np} \le C(n,p)(\mathrm{diam}\ B)\left(\fint_B |\nabla u|^p dx\right)^{1/p} \tag{1.2}$$

は，**ソボレフ-ポアンカレの不等式**と呼ばれる。ここに，$1 \le p < n$ である。

定理 1.1 （ソボレフの定理）　$p \ge 1$，$u \in W^{1,p}(\mathbf{R}^n)$ とすると，

(1) $1/q > 1/p - 1/n \ge 0$ ならば，$u \in L^q_{loc}(\mathbf{R}^n)$

(2) $1/q = 1/p - 1/n > 0$ ならば，$u \in L^q(\mathbf{R}^n)$

(3) $p > n$ ならば，u は任意の球 B 上ヘルダー連続で

$$|u(x) - u(x')| \le C|x - x'|^{1-n/p}\left(\int_B |\nabla u|^p dy\right)^{1/p} \qquad (\forall x, x' \in B) \tag{1.3}$$

$p = n$ のとき，定理 1.1 の (1) と (3) の間を埋めるような結果も知られている。すなわち，

$$\int_B |\nabla u|^n \varphi(|\nabla u|) dx < \infty \tag{1.4}$$

を満たすような（**ソボレフ-オーリッツ関数**）u について，φ の増大度に応じて (1)，(2) または (3) のタイプの性質が成り立つことが示される (cf. [12])。

2 距離空間上のソボレフ関数

(X,d) を距離空間とし，その上のボレル測度 μ を考える。測度 μ は次の条件を満たすものとする：ボレル集合 A とそれに含まれる球 B に対して，

$$\frac{\mu(B)}{\mu(A)} \geq C\left(\frac{\text{diam } B}{\text{diam } A}\right)^Q \tag{2.1}$$

が成り立つ。ここに，Q と C は正の定数である。とくに，μ は **doubling** である：

$$\mu(B) \leq \mu(2B) \leq C\mu(B) \tag{2.2}$$

ここに，$B = B(x,r)$，$2B = B(x,2r)$，C は正の定数である。

X 上のボレル関数の組 (u,g) が **p-ポアンカレ不等式**を満たすとは，任意の球 B に対して，

$$⨍_B |u - u_B| d\mu \leq C(\text{diam } B)\left(⨍_{\sigma B} g^p d\mu\right)^{1/p} \tag{2.3}$$

が成り立つ。ここに，g は非負，$\sigma \geq 1$，C は正の定数で $p > 0$ である。(2.3) と次は同値である。

$$\inf_c ⨍_B |u - c| d\mu \leq C(\text{diam } B)\left(⨍_{\sigma B} g^p d\mu\right)^{1/p} \tag{2.4}$$

注 2.1 (1) $0 < p < 1$ のとき，(2.3) を満たさないソボレフ関数が存在する。

(2) 調和関数はすべての $p > 0$ に対して (2.3) を満足する。

定理 2.1 (cf. [4]) $1 \leq p < Q$ とする。(u,g) が p-ポアンカレ不等式を満たすならば，$0 < q < pQ/(Q-p)$ に対して，

$$\left(⨍_B |u - u_B|^q d\mu\right)^{1/q} \leq C(\text{diam } B)\left(⨍_{5\sigma B} g^p d\mu\right)^{1/p} \tag{2.5}$$

注 2.2 (1) 定理 2.1 において，さらに "truncation property" を仮定すれば，$q = pQ/(Q-p)$ とすることもできる。

(2) 定理 2.1 において，球でうまく覆える領域 A に置きかえて，不等式

$$\left(\fint_A |u-u_A|^q d\mu\right)^{1/q} \le C(\mathrm{diam}\ A)\left(\fint_A g^p d\mu\right)^{1/p} \tag{2.6}$$

を得ることが可能である。ここで，雑にいうなら，A は境界がきれいな集合ということもできる。実際には，次のような領域なら O.K. である。

定義 2.1　$\lambda \ge 1$, $M \ge 1$, $a > 1$ とする。有界集合 A が（球 B_0 に関して）(λ, M, a)**-連結条件**を満たすとは，任意の $x \in A$ に対して，球の列 $\{B_1, B_2, B_3, ...\}$ で次の条件を満たすものが存在する：

(i) $\lambda B_i \subset A$　　　$(\forall i \ge 0)$

(ii) $x \in B_i$　　　(i：十分大)

(iii) $M^{-1}a^{-i}\mathrm{diam}\ A \le \mathrm{diam}\ B_i \le Ma^{-i}\mathrm{diam}\ A$　　　$(\forall i \ge 0)$

(iv) $\exists B_i'$：球 s.t. $B_i' \subset B_i \cap B_{i+1}$ かつ $B_i \cup B_{i+1} \subset MB_i'$　　　$(\forall i \ge 0)$

(2.6) の証明には，次の補題を利用する。

補題 2.1　$\alpha > 0$ で

$$\mu(\{x \in X : |u(x)| > t\}) \le C_0 t^{-\alpha} \tag{2.7}$$

ならば，$0 < q < \alpha$ に対して

$$\|u\|_q \le 2^{1/q}\left(\frac{\alpha C_0}{\alpha - q}\right)^{1/\alpha} \mu(X)^{(\alpha-q)/\alpha q} \tag{2.8}$$

$p \ge Q$ のときには，次の結果が知られている。

定理 2.2（Trudinger の不等式）　X は連結で $Q > 1$ とする。関数組 (u, g) が Q-ポアンカレ不等式を満たすならば，定数 C_1, C_2 で次の条件を満たすものが存在する：任意の球 B に対して

$$\fint_B \exp\left(\frac{C_1\mu(B)^{1/Q}|u-u_B|}{(\mathrm{diam}\ B)\|g\|_Q}\right)^{Q/(Q-1)} d\mu \le C_2 \tag{2.9}$$