Computational Commutative Algebra and Combinatorics

ADVANCED STUDIES
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Computational Commutative
Algebra and Combinatorics

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Foreword

When the first international conference on commutative algebra and combinatorics was held in 1985 in Kyoto, the connections between combinatorics and commutative algebra were just becoming established. At this meeting I had the pleasure of meeting a recently graduated student of Professor Hideyuki Matsumura named Takayuki Hibi. One could not fail to admire the tremendous enthusiasm and energy which he displayed toward all aspects of mathematics, from his own research to the details of keeping the conference running smoothly. The conference turned out to be a tremendous success and opened up many doorways to future collaboration between western and Japanese mathematicians in the exciting new area of "combinatorics and commutative algebra." No one has been a more tireless exponent of promoting this field than Takayuki Hibi. In particular, in addition to his prolific and original research he has been the main organizer of two subsequent highly successful meetings in Japan on combinatorics and commutative algebra.

The present volume constitutes the proceedings of the second of these two meetings and confirms that the subject has lost none of its original vitality. New techniques such as toric varieties and Gröbner bases have greatly expanded the connections between combinatorics and commutative algebra. For instance, there is now a well-developed theory of the set of all triangulations of a set of points in Euclidean space that uses sophisticated recent developments in both commutative algebra and combinatorics. A further important development of the subject is the introduction of exterior algebra techniques, especially the theory of algebraic shifting pioneered by Gil Kalai. His contribution to the present volume should become the definitive overview of this theory. Just a glance through the titles of the papers contained here will show the extent to which commutative algebra has pervaded other areas of mathematics. Readers of all or part of these proceedings owe a debt of gratitude to Takayuki Hibi and his students Hidefumi Ohsugi and Tomonori Kitamura for bringing together a plethora of diverse experts in commutative algebra to Osaka and for arranging for so many of them to contribute high-quality articles to these proceedings. I look forward to many future beautiful results stimulated by the contents of this volume.

18 April 2001
Richard Stanley
Cambridge, Massachusetts

Preface

The 8th Mathematical Society of Japan International Research Institute "Computational Commutative Algebra and Combinatorics" was held in Osaka University, Toyonaka Campus, Osaka, Japan, July 21 – July 30, 1999. The organizing committee consisted of Jürgen Herzog, Takayuki Hibi and Richard Stanley. The present volume fills the role of the proceedings of the meeting.

The meeting was devoted to informing participants of recent progress and to stimulating new research in the computational aspects of combinatorics, commutative algebra, and related fields. Topics discussed included *Hilbert functions, generic initial ideals, Gröbner bases, algebraic shifting, toric varieties, convex polytopes, triangulations, and partially ordered sets*. More than a hundred participants, including 26 international participants from Bulgaria, Canada, Germany, India, Sweden, US, and Vietnam, attended the meeting.

Part I of the meeting, July 21 – 24, consisted of expository lecture series for graduate students given by the five speakers, Jürgen Herzog, Gil Kalai, Claudio Procesi, Vasudevan Srinivas, and Richard Stanley. Each speaker gave three one-hour lectures. Part II of the meeting, July 26 – 30, consisted of 22 one-hour invited research talks.

The fascinating research area "combinatorics and commutative algebra" originates from a pioneered work by Richard Stanley in 1975 on a proof of the Upper Bound Conjecture for spheres by means of the theory of Cohen–Macaulay rings. Since 1980, the geometry of toric variety has played important roles to develop combinatorics on convex polytopes. Moreover, in the early 90's, the technique on Gröbner bases turned out to be indispensable to study discrete structures on convex polytopes.

In Japan, there had been two international conferences on commutative algebra and combinatorics in the past; the US–Japan joint seminar on commutative algebra and combinatorics, Kyoto, 1985, and the ICM 90 satellite conference on commutative algebra and combinatorics, Nagoya, 1990. The proceedings of the first one were published in this series of the Advanced Studies in Pure Mathematics (volume 11, "Commutative Algebra and Combinatorics" (H. Matsumura and M. Nagata, Eds.), 1987). We hope the present volume will stimulate further developments in the computational aspects of combinatorics and commutative algebra.

We wish to thank the Mathematical society of Japan for choosing our research plan to be the topic of 1999 International Research Institute

and to thank the Japanese Associations of Mathematical Sciences for financial support to invite young researchers from several countries.

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1 May 2001
Takayuki Hibi
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