Counting the number of primitive idempotents of cyclotomic KLR algebras of type $A_n^{(1)}$

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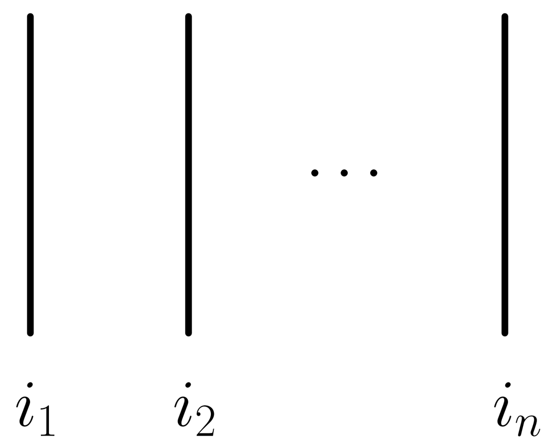
• What's the KLR algebra?

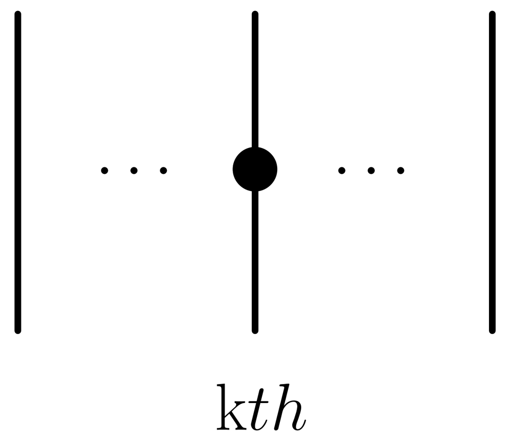
Roughly speaking, its basis are diagrams, generated by three diagrams and identified by local relations.
Multiplication is defined like braid groups, but we must check the colors are the same or not.

Three basic diagrams

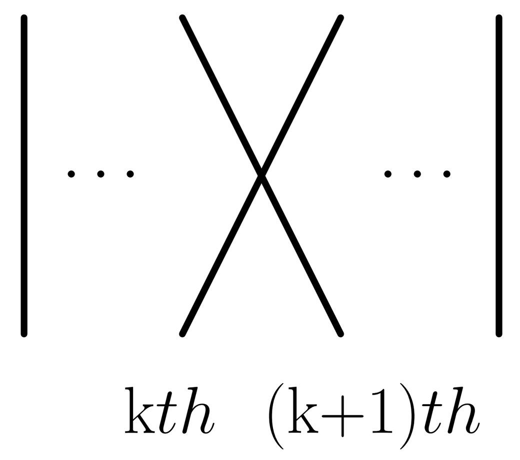
1.KLR idempotents : $\mathbf{e}(\mathbf{i})$
For $\mathbf{i} = (i_1, i_2, \cdots, i_n)$, which is permutation of $(1, 2, \cdots, n)$, the color of each strands are determined from left to right.



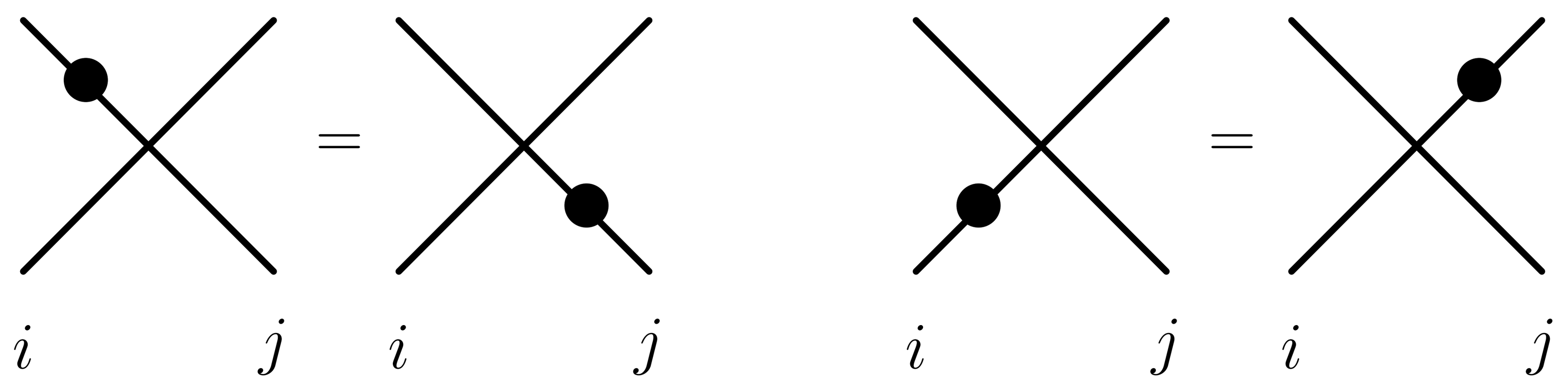
2.Dot : y_k $(1 \leq k \leq n)$
Dots can be put on strands. These strands are not colored so this diagram connect with any diagrams.

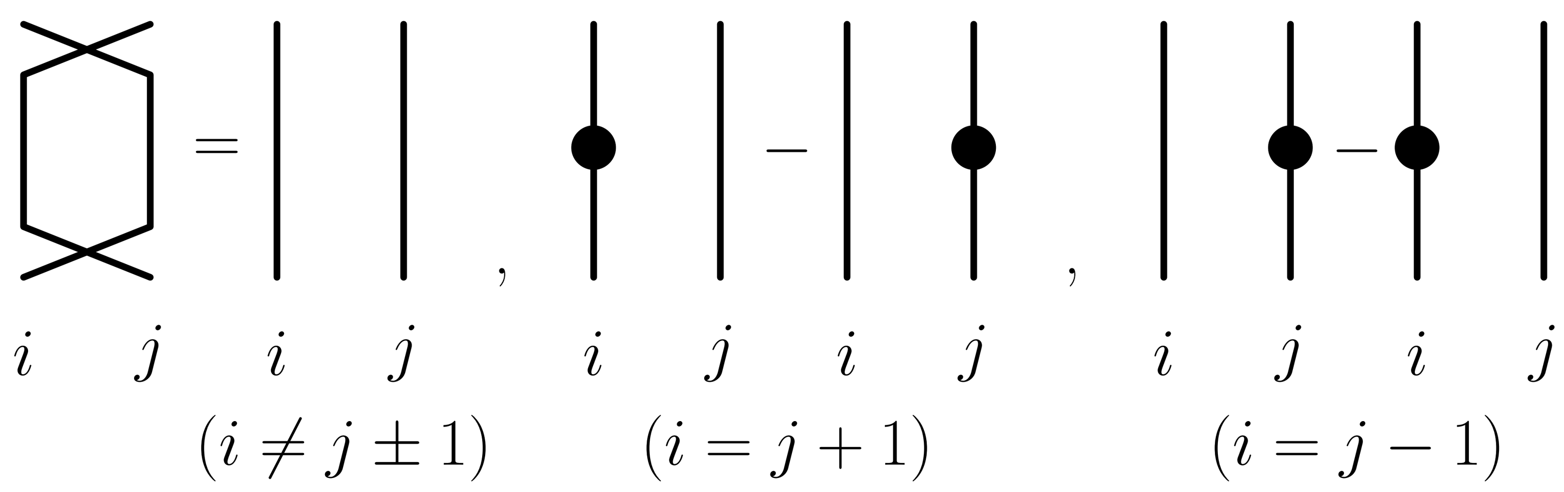


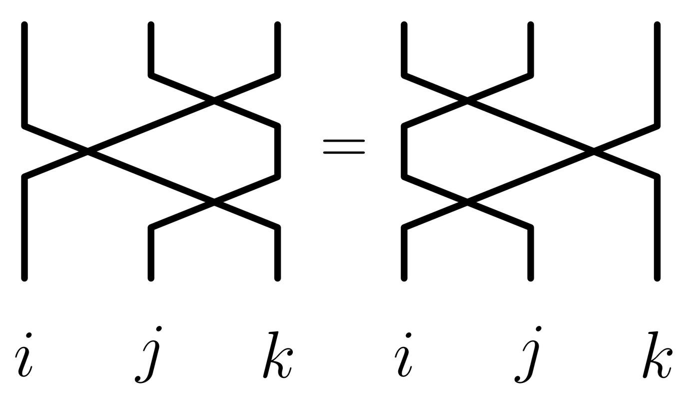
3.Cross : $\psi_k (1 \leq k < n)$
Strands can cross each other. This diagram connect with any diagrams as well.



Local relations

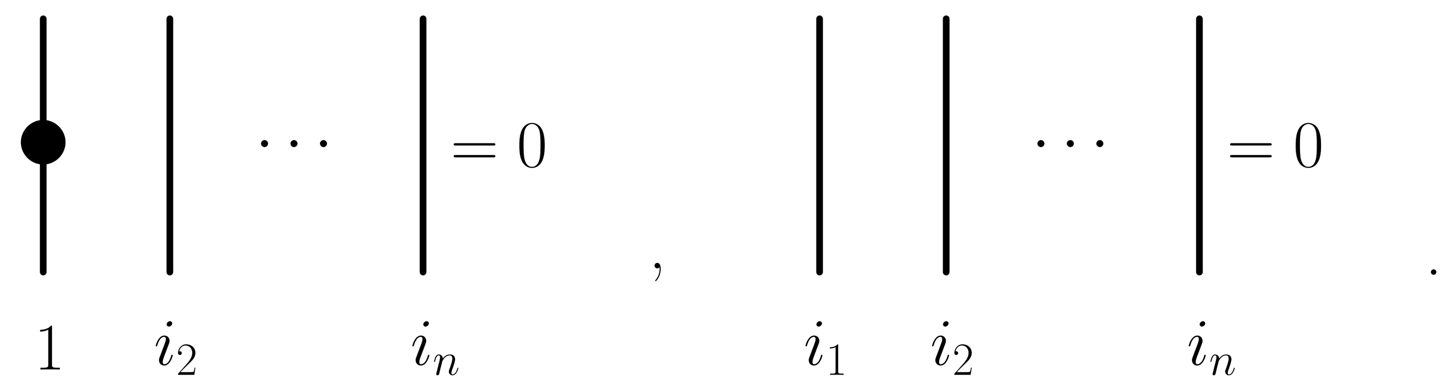






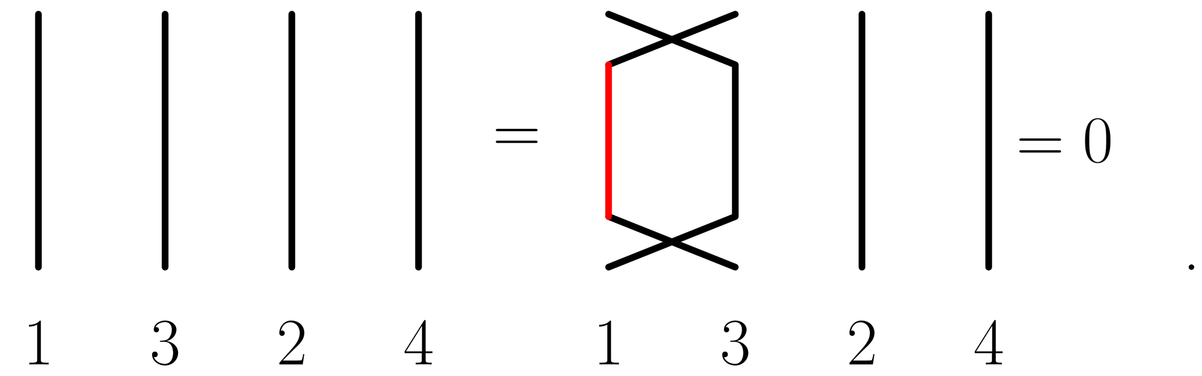
Cyclotomic ideal

Cyclotomic ideal is generated by following relations.
We define cyclotomic KLR algebra as a quotient of KLR algebra by the ideal.



Vanishment of KLR idempotents

Clearly, KLR idempotents vanish if $i_1 \neq 1$, but there is another case in which also KLR idempotents vanish. For example, $n = 4$,



Then, there is a natural question.

Question

How much KLR idempotents can remain?

Answer$(+\alpha)$

There are 2^{n-2} KLR idempotents which don't vanish. Moreover, they all are primitive.

The 2^{n-2} KLR idempotents are constructed explicitly, and it's easy to show that the others vanish: