The 1st MSJ-SI

The Mathematical Society of Japan, Seasonal Institute

Probabilistic Approach to Geometry

This program is partially supported by International Scientific Meetings in Japan, JSPS.

Date : July 28 – August 8, 2008

Venue : Kyoto University, Kyoto, Japan

The first week (July 28th – August 1st)
Kyoto University Clock Tower Centennial Hall

The second week (August 4th – August 8th)
Shiran Kaikan

Organizing Committee

Motoko Kotani (Chair), Tohoku University
Hitoshi Arai, University of Tokyo
Takashi Kumagai, Kyoto University
Karl-Theodor Sturm, Universität Bonn

Local Organizing Committee

Takashi Kumagai (Chair), Kyoto University
Masanori Hino, Kyoto University
Tsuyoshi Kato, Kyoto University
Shin-ichi Ohta, Kyoto University

Greeting

Welcome to the first MSJ-SI workshop 'Probabilistic Approach to Geometry' !

The Mathematical Society of Japan, Seasonal Institute (MSJ-SI in short) is a series of workshops hosted by the Mathematical Society of Japan. It will be held once every year starting this year 2008. The second MSJ-SI 'Arrangements of Hyperplanes' is planned to be held in August 2009 at Hokkaido University.

Our idea is to obtain an overview of the significant recent developments in the subject, and to present it especially to young researchers in Japan, as well as Asian countries. The subject is chosen by the Scientific Committee of the Mathematical Society of Japan every year. This is a succession of our conference series Mathematical Society of Japan, International Research Institute (MSJ-IRI in short). We had 15 successful conferences, and our idea is to continue the conference in the same spirit, but also to contribute mathematical communities outside Japan.

The Scientific Committee chose the theme 'Probabilistic Approach to Geometry' about two years ago, and asked to Professor Motoko Kotani to form an organizing committee. She and other committee members have worked hard since then, and I am grateful for them to turn our idea to the actual workshop.

Finally I would like take this opportunity to thank all lecturers, speakers, participants, staffs who contribute the workshop in various ways. We hope that the MSJ-SI will be successful and bring future progress in mathematics.

Hiraku Nakajima
The Former Chair of The Scientific Committee,
Mathematical Society of Japan

Survey Lecturers

Shigeki Aida
(Osaka University)

Terry J. Lyons
(University of Oxford)

Yann Ollivier
(École Normale Supérieure de Lyon)

Laurent Saloff-Coste
(Cornell University)

Cédric Villani
(École Normale Supérieure de Lyon)

Invited Speakers

Shun-ichi Amari
(Riken)

Atsushi Atsuji
(Keio University)

Dominique Bakry
(Université Paul Sabatier)

Martin T. Barlow
(University of British Columbia)

Shinto Eguchi
(The Institute of Statistical Mathematics)

Kenneth David Elworthy
(University of Warwick)

Roland Friedrich
(Max-Planck-Institute für Mathematik)

Keisuke Hara
(Ritsumeikan University)

Yuzuru Inahama
(Tokyo Institute of Technology)

Hiroyasu Izeki
(Tohoku University)

Vadim Kaimanovich
(Jacobs University Bremen)

Atsushi Kasue
(Kanazawa University)

Tsuyoshi Kato
(Kyoto University)

Yoshikata Kida
(Tohoku University)

Jun Kigami
(Kyoto University)

Chang-Wan Kim
(Korea Institute for Advanced Study)

Takefumi Kondo
(Kyoto University)

Kazumasa Kuwada
(Ochanomizu University)

Kazuhiro Kuwae
(Kumamoto University)

John Lott
(University of Michigan)

Hiroshi Matsuzoe
(Nagoya Institute of Technology)

Robert J. McCann
(University of Toronto)

Shin-ichi Ohta
(Kyoto University)

Yukio Otsu
(Kyushu University)

Vladimir Pestov
(University of Ottawa)

Ichiro Shigekawa
(Kyoto University)

Takashi Shioya
(Tohoku University)

Tatsuya Tate
(Nagoya University)

Anton Thalmaier
(Université du Luxembourg)

Bálint Virág
(University of Toronto)

Max von Renesse
(Technische Universität Berlin)

Sumio Watanabe
(Tokyo Institute of Technology)

Andrzej Zuk
(Université Paris 7)

Program

The first week (July 28th – August 1st)
Kyoto University Clock Tower Centennial Hall

July 28th (Mon.)

9:45–10:15	Registration
10:15–10:30	**President of the Mathematical Society of Japan** Opening remark
10:30–11:20	**Cédric Villani**, École Normale Supérieure de Lyon, Lecture 1 Optimal transport in geometry
11:20–11:40	Tea
11:40–12:30	**Terry J. Lyons**, University of Oxford, Lecture 1 Rough paths — A story in non-commutative analysis
12:30–14:00	Lunch
14:00–14:50	**Martin T. Barlow**, University of British Columbia Uniqueness of Brownian motion on the Sierpinski carpet
15:00–15:30	**Max von Renesse**, Technische Universität Berlin Entropic measure and Wasserstein diffusion
15:40–16:10	**Tsuyoshi Kato**, Kyoto University A dynamical pattern formation, tropical geometry and informative entropy
16:10–16:40	Tea
16:40–17:30	**Roland Friedrich**, Max-Planck-Institute für Mathematik The global geometry of stochastic Loewner evolutions

July 29th (Tue.)

10:00–10:50	**Cédric Villani**, École Normale Supérieure de Lyon, Lecture 2 Optimal transport in geometry
10:50–11:10	Tea
11:10–12:00	**Shigeki Aida**, Osaka University, Lecture 1 Rough path analysis: An introduction
12:00–14:00	Lunch
14:00–14:50	**Bálint Virág**, University of Toronto Random matrices, probability, and geometry
15:00–15:30	**Tatsuya Tate**, Nagoya University Bernstein measures on convex polytopes
15:40–16:10	**Chang-Wan Kim**, Korea Institute for Advanced Study Ricci and flag curvatures in Finsler geometry
16:10–16:40	Tea
16:40–17:30	**Shin-ichi Ohta**, Kyoto University Optimal transport and Ricci curvature in Finsler geometry

July 30th (Wed.)

10:00–10:50	**Shun-ichi Amari**, Riken Information geometry, its applications and related mathematical problems
10:50–11:10	Tea
11:10–12:00	**Shigeki Aida**, Osaka University, Lecture 2 Rough path analysis: An introduction
12:00–14:00	Lunch
14:00–14:50	**Sumio Watanabe**, Tokyo Institute of Technology What we can estimate about a singularity from random samples
15:00–15:30	**Hiroshi Matsuzoe**, Nagoya Institute of Technology Statistical manifolds and affine differential geometry
15:40–16:10	**Kazuhiro Kuwae**, Kumamoto University On discrete harmonic maps into CAT(k)-spaces via Markov chains
16:10–16:40	Tea
[Poster]	**Hyun Yoo**, Hankyong National University Projections in the reproducing kernel Hilbert spaces and the conditional probabilities of determinantal point processes in discrete spaces
[Poster]	**Wen-Haw Chen**, Tunghai University On topological obstructions of compact Riemannian and combinatorial positively Ricci curved manifolds
16:40–17:30	**Shinto Eguchi**, The Institute of Statistical Mathematics Information divergence geometry and its application to machine learning
18:30–20:30	Buffet-style party (Kyodai-Kaikan)

July 31st (Thu.)

10:00–10:50	**Cédric Villani**, École Normale Supérieure de Lyon, Lecture 3 Optimal transport in geometry
10:50–11:10	Tea
11:10–12:00	**Shigeki Aida**, Osaka University, Lecture 3 Rough path analysis: An introduction
12:00–14:00	Lunch
14:00–14:50	**Takashi Shioya**, Tohoku University Geometric analysis on Alexandrov spaces
15:00–15:30	**Yuzuru Inahama**, Tokyo Institute of Technology A stochastic Taylor-like expansion in the rough path theory
15:40–16:10	**Kazumasa Kuwada**, Ochanomizu University Characterization of maximal Markovian couplings for diffusion processes
16:10–16:40	Tea
16:40–17:30	**Keisuke Hara**, Ritsumeikan University Rough path condition for smooth paths

August 1st (Fri.)

10:00–10:50	**Cédric Villani**, École Normale Supérieure de Lyon, Lecture 4 Optimal transport in geometry
10:50–11:10	Tea
11:10–12:00	**Vladimir Pestov**, University of Ottawa Urysohn's universal, or random, metric space, its group of isometries, and other related structures
12:00–14:00	Lunch
14:00–14:50	**Robert J. McCann**, University of Toronto Curvature, continuity and uniqueness of optimal transportation maps
15:00–16:10	Contributed Talks
	Kouji Yano, Kobe University Excursions away from a regular point for one-dimensional symmetric Lévy processes without Gaussian part
	Hiroshi Kawabi, Okayama University Riesz transforms on a path space with Gibbs measures
	AbdulRahman Al-Hussein, Qassim University Time-dependent backward stochastic evolution equations
16:10–16:40	Tea
16:40–17:30	**Yukio Otsu**, Kyushu University Statistical mechanics of 1-particle ideal gas and deformation of Alexandrov spaces

August 2nd (Sat.)

Excursion	**13:30–17:30**

The second week (August 4th – August 8th)
Shiran Kaikan

August 4th (Mon.)

10:00–10:50	**Laurent Saloff-Coste**, Cornell University, Lecture 1 Heat kernel estimates
10:50–11:10	Tea
11:10–12:00	**Yann Ollivier**, École Normale Supérieure de Lyon, Lecture 1 Survey on random groups
12:00–14:00	Lunch
14:00–14:50	**John Lott**, University of Michigan Optimal transport and Perelman's reduced volume
15:00–15:30	**Yoshikata Kida**, Tohoku University Orbit equivalence rigidity for some groups acting on trees
15:40–16:10	**Atsushi Atsuji**, Keio University Estimates on the number of omitted values of meromorphic functions
16:10–16:40	Tea
16:40–17:30	**Vadim Kaimanovich**, Jacobs University Bremen Random graphs and equivalence relations

August 5th (Tue.)

10:00–10:50	**Laurent Saloff-Coste**, Cornell University, Lecture 2 Heat kernel estimates
10:50–11:10	Tea
11:10–12:00	**Yann Ollivier**, École Normale Supérieure de Lyon, Lecture 2 Discrete positive curvature, Markov chains and concentration of measure
12:00–14:00	Lunch
14:00–14:50	**Terry J. Lyons**, University of Oxford, Lecture 2 Rough paths — A story in non-commutative analysis
15:00–16:10	Contributed Talks
	Masayoshi Watanabe, Tohoku University Concentration of measure via approximated Brunn-Minkowski inequalities
	Kei Funano, Tohoku University Concentration of 1-Lipschitz maps and group action
	Asuka Takatsu, Tohoku University On Wasserstein geometry of the space of Gaussian measures
16:10–16:40	Tea
16:40–17:30	**Hiroyasu Izeki**, Tohoku University A fixed-point property of discrete groups and an energy of equivariant maps

August 6th (Wed.)

10:00–10:50	**Laurent Saloff-Coste**, Cornell University, Lecture 3 Heat kernel estimates
10:50–11:10	Tea
11:10–12:00	**Yann Ollivier**, École Normale Supérieure de Lyon, Lecture 3 Discrete positive curvature, Markov chains and concentration of measure
12:00–14:00	Lunch
14:00–14:50	**Anton Thalmaier**, Université du Luxembourg Li-Yau type inequalities and a priori estimates for heat equations by stochastic analysis
15:00–15:30	**Takefumi Kondo**, Kyoto University Fixed-point property of random groups
15:40–16:10	**Jun Kigami**, Kyoto University Measurable Riemannian geometry on the Sierpinski gasket
16:10–16:30	Tea
16:30–17:10	Contributed Talks
	Naotaka Kajino, Kyoto University Weyl type spectral asymptotics for the Laplacian on Sierpinski carpets
	Ryoki Fukushima, Kyoto University Brownian survival among perturbed lattice traps
19:00–21:00	Banquet (Ganko Takasegawa Nijoen)

August 7th (Thu.)

10:00–10:50	**Laurent Saloff-Coste**, Cornell University, Lecture 4 Heat kernel estimates
10:50–11:10	Tea
11:10–12:00	**Yann Ollivier**, École Normale Supérieure de Lyon, Lecture 4 Discrete positive curvature, Markov chains and concentration of measure
12:00–14:00	Lunch
14:00–14:50	**Dominique Bakry**, Université Paul Sabatier Gradient bounds for some hypo-elliptic heat equations
15:00–16:10	Contributed Talks
	Juillet Nicolas, University of Bonn Synthetic Ricci curvature bounds in the Heisenberg group
	Takumi Yokota, University of Tsukuba Perelman's reduced volume and gap theorem for the Ricci flow
	Shinichiroh Matsuo, University of Tokyo The Runege theorem for instantons
16:10–16:40	Tea
16:40–17:30	**Atsushi Kasue**, Kanazawa University Functions of finite Dirichlet sum and compactifications of infinite graphs

August 8th (Fri.)

10:00–10:50	**Ichiro Shigekawa**, Kyoto University Non symmetric diffusions on a Riemannian manifold
10:50–11:10	Tea
11:10–12:00	**Terry J. Lyons**, University of Oxford, Lecture 3 Rough paths — A story in non-commutative analysis
12:00–14:00	Lunch
14:00–14:50	**Andrzej Zuk**, Université Paris 7 Automata groups
15:00–15:50	**Kenneth David Elworthy**, University of Warwick Stochastic flows and geometric analysis on path spaces

Abstracts

Survey Lectures

Rough path analysis: An Introduction

Shigeki Aida

Osaka University

Analysis of functional of stochastic processes is an important subject in probability theory. Typical functionals are obtained by solutions of stochastic differential equations. Since the driving path is not smooth, we need to make clear the meaning of solutions.

This is usually carried out by using Ito's stochastic analysis and martingale theory. However, recently, Terry Lyons introduced the notion of rough path and present a new approach to the problem.

He can prove the continuity property of solutions of SDE with respect to the driving paths. The important point is that the rough path theory is formulated independently of probability theory and estimates in probability theory are used to apply rough path theory to studies on stochastic equations. Therefore estimates in stochastic analysis are still necessary in this sense. The aim of this lecture is to give an introduction of rough path theory to the participants who may be not familiar with probability theory and the rough plan of this talk is as follows:

I A continuity theorem on integration of one forms
II Introduction of the notion of rough paths and main theorems
III Brownian rough paths
IV Relation to Stochastic integrals and SDE
V Applications

Rough Paths — A story in non-commutative analysis

Terry J. Lyons

University of Oxford

Systems that evolve and interact play an important part in many aspects the world as we understand it, and on many scales. Powerful mathematical tools have been developed to model and predict the behaviour of these systems; with Newtonian calculus at the centre.

In many contexts, the systems that interact are complex, highly oscillatory, and on normal scales, are far from differentiable. For this reason it is necessary to take calculus forward and develop a mathematical theory for differential equations that couple systems of a much broader kind if that is indeed possible.

Itô made just such a large step in 1942, when he explained how equations such as $dy_t = y_t dx_t, y_0 = 1$ could be made rigorous when the input x_t is almost any Brownian path (one knows that almost every Brownian path has no points of increase); taking the method forward, and treating the non-linear and vector case $dy_t = f(y_t)dx_t, y_0 = a$ Itô gave us a tool that was to have unbelievably wide ranging consequences.

However, the solutions constructed by Itô were, ab initio, random variables with values in path space, and were constructed using L^2 arguments. In particular, solutions could be redefined on any given path and they would still be solutions. The driving noise x needed to be a semi-martingale.

The theory of rough paths revisits the underlying theory of differential equations and, building on work of the geometer KT Chen and the analyst LC Young, it has proved possible to develop a well defined deterministic theory rich enough to recover the main results relating to stochastic differential equations, and indeed extend them so that they apply to many processes (such as fractional Brownian motion) that cannot be treated by Itô's theory.

At a pure mathematical level, Itô's theory has lead to many deep theorems, Malliavin's proof of Hörmander's theorem is a very good example. Recently Cass and Friz have extended Malliavin's result to prove the existence of a density for the law of a diffusion driven by a fractional Brownian motion.

Key to this development of rough paths is the identification of structured and quantitative ways to describe path segments in a top down way. The descriptions should capture the most significant features of a path segment $x_u, u \in [s,t]$ so that one can predict its effect. Newtonian calculus is premised on the idea that a chord is a good approximation to a path; a smooth path is one well approximated by its chords. But a chord is a very commutative description!

Paths (in a vector space) have an operation (concatenation) and this operation is essentially non-commutative. The mapping to the chord is a homomorphism from path segments into the abelian group $\mathbb{R}^d$. By replacing the abelian chordal description of a path segment with a finite dimensional non-commutative description of the segment one is able to develop a much richer picture.

The first lecture will develop some of these threads. The last two lectures will be independent, and will look at applications of rough paths and at the mathematical properties of the signature of a path.

Lecture 1: Survey on Random groups
Lecture 2-4: Discrete positive curvature, Markov chains and concentration of measure

Yann Ollivier

École Normale Supérieure de Lyon

In my surveys I will present two different fields in which geometry and probability interact: 1) Concentration of measure 2) Random groups. These can be seen as pertaining, respectively, to positive-curvature or negative-curvature geometry.

Concentration of measure is a phenomenon which forces functions on a given space to be "almost constant". From the probabilistic viewpoint it can be seen as a generalization of the law of large numbers, whereas from the geometric viewpoint is has deep connections with positive (Ricci) curvature. I will present the basic ideas behind this phenomenon and introduce a common framework based on curvature to deal with both continuous and discrete examples.

Random groups have been introduced by M. Gromov to give a formal meaning to the statement that "a typical infinite group has negative curvature". They can also be used together with other ingredients to construct "wild" groups with new properties. I will give an overview of the various notions of "random groups" together with their main properties.

Heat kernel estimates

Laurent Saloff-Coste

Cornell University

In this lectures, I will review some of the techniques that have been developed to obtain good heat kernel estimates. I will also dicuss some apllications of tehse estimates.

Optimal transport in geometry

Cédric Villani

École Normale Supérieure de Lyon

In a series of lectures I shall describe the relations of optimal transport with various notions of curvature (Ricci curvature; sectional curvature and generalizations thereof).

The treatment will be inspired from my recent book, ”Optimal transport, old and new”.

Invited Talks

Information Geometry, its Applications and Related Mathematical Problems

Shun-ichi Amari

RIKEN Brain Science Institute

Information geometry emerged from studies of invariant structure in a manifold of probability distributions. It has a Riemann metric and dually coupled affine connections in this metric. Given a family of probability distributions p(x, u) where x is a random variable, u is a vector parameter to specify the distributions, we consider a manifold consisting of all such distributions such that u plays a role of a permissible coordinate system. We require the following invariance:

1) The geometry should be invariant under coordinate transformations.
2) The geometry should be invariant under transformations of random variable x.

We then have a unique invariant Riemannian metric, which turns out to be the Fisher information matrix, and a one-parameter family of affine connections including the Levi-Civita connection as a special case.

We present a concise introduction to information geometry, and explain its properties. The dual geometry includes affine differential geometry and Hessian manifolds as special cases. In particular, when a manifold is dually flat with non-vanishing Riemann-Christoffel curvature (with respect to Levi-Civita connection), the space is closely connected with the Legendre duality structure together with a pair of convex functions.

We explain applications of dual geometry to various fields of science such as statistical inference, machine learning, convex optimization, and neural networks. We finally present some mathematical problems to be solved in relation to the dual geometry. Some of the problems are as follows:

1) Given a Riemannian manifold with dual affine connections, what conditions guarantee that the manifold can be naturally embedded in a higher-dimensional dually flat manifold of finite dimensions?
2) Given a Riemannian manifold, what conditions guarantee that it has a dual pair of affine connections in relation to the metric?
3) Given a Riemannian manifold with dual affine connections, what conditions guarantee that it is equivalent to the manifold of a family of probability distributions?

Estimates on the number of omitted values of meromorphic functions

Atsushi Atsuji

Keio University

We discuss some estimates on the number of omitted values of meromorphic functions on Koehler manifolds. We use stochastic calculus to estimate the number, establishing an analogy of Nevanlinna theory for meromorphic functions on general Koehler manifolds.

Gradient bounds for some hypo-elliptic heat equations

Dominique Bakry

Université Paul Sabatier, Toulouse, France

For heat equations associated to elliptic operators, many efficient controls of the gradient of the solution at time t may be obtained from the gradient at time 0 (what we call gradient bounds) using the notion of Ricci curvature. Theses bounds do not extend to even the simplest hypo-elliptic systems like the Kohn Laplacian on the Heisenberg group. Nevertheless, some recent works show that with some effort, one may still get gradient bounds which are similar to the elliptic ones.

We shall present here different approach to those bounds, on simple groups like Heisenberg or SU(2), ore more general hypo-elliptic systems.

Uniqueness of Brownian motion on the Sierpinski carpet

Martin T. Barlow

University of British Columbia

The Sierpinski carpet (SC) is a fractal subset of R^d, and has proved a useful and challenging 'model space' for the development of techniques related to diffusions and heat kernels on irregular media and metric measure spaces.

While the existence of well-behaved limiting diffusions on the SC have been established, two main problems were left open. The first was the uniqueness of the process, and the second the characterisation of the space-time scaling parameter d_w.

The SC is not finitely ramified, so the finite dimensional methods that can be used on some other families of fractals cannot be used.

In this talk I will describe recent progress on the first problem. This is joint work with R.F. Bass, T. Kumagai and A. Teplyaev.

Information divergence geometry and its application to machine learning

Shinto Eguchi

The Institute of Statistical Mathematics

In this talk we consider a problem on which geometry is appropriate to understand a space of probability measures in a deeper manner. One promising approach has been progressed in 'information geometry' which is advocated by Amari in 1990s on the basis of his fundamental work on statistics and neural network going back Rao's observation (1945) of a statistical model M as a Riemannian space by the information metric g. Information geometry defines the mixture connection ∇^{m} and exponential connection ∇^{e} from an invariance point of view. We discuss an information divergence on a space P of probability density functions including Kullback-Leibler divergence which is closely related with Boltzmann-Shannon entropy. An information divergence D is defined by the first axiom of distance, that is $D(p,q) \geq 0$ with equality iff $p = q$. Let M be a differentiable manifold in P. Then D leads to Riemannian metric g^D and two linear connections ∇^D and ${}^*\nabla^D$ on M by Taylor approximation, cf. Eguchi (1983). It is shown that ∇^D and ${}^*\nabla^D$ are conjugate with respect to g^D, that is,

$$Xg^D(Y,Z) = g^D(\nabla^D_X Y, Z) + g^D(Y, {}^*\nabla^D_X Z).$$

Let U be a convex function with positive derivative. From convex duality theorem we know the conjugate convex function U^* defined by

$$U^*(s) = \max_{-\infty<t<\infty}\{ts - U(t)).$$

Note that $\xi(s) = \arg\max_t\{ts - U(t))$ is the derivative of $U^*(s)$. Thus, U-diagonal entropy on P is defined by $H_U(p) = \int U^*(p)$ and U-diagonal entropy is defined by $C_U(p,q) = \int\{p\xi(q) - U(\xi(q))\}$. We define U-divergence D_U by the difference of U-diagonal entropy with U-cross entropy, that is $D_U(p,q) = H_U(p) - C_U(p,q)$. By definition D_U satisfies the first axiom of distance. If $U(t) = \exp(t)$, then $U^*(s) = s\log(s) - s$, $\xi(s) = \log(s)$, which laeds to Kullback-Leibler divergence $D_{\exp}(p,q) = \int\{q - p - p(\log(q) - \log(p))\}$. Obviously, we know many other convex functions other than the exponential function. For example, a power-exponential function $U(t) = (1+\beta t)^{1/\beta+1}/(\beta+1)$ leads to the conjugate function $U^*(s) = s^{\beta+1}/\beta(\beta+1) - s/\beta$, so that we get the power divergence

$$D_\beta(p,q) = \int\{(p^{\beta+1} - q^{\beta+1})/(\beta+1) - p(p^\beta - q^\beta)/\beta\},$$

which is closely related with Tsallis entropy. A direct discussion on U-divergence yields an interesting characteristic such that the linear connection ∇^{D_U} is the mixture connection ∇^{m} for any U. This characteristic is shown to be helpful for applications in machine learning context. In a final part we briefly introduce applications of U-entropy and U-divergence for proposing learning algorithms in statistical machine learning beyond maximum likelihood principle, which include independent component analysis and statistical pattern recognition.

Stochastic Flows and Geometric Analysis on Path Spaces

Kenneth David Elworthy

Mathematics Institute, University of Warwick

This will review some geometry and analysis on the space of based continuous paths C_xM on a compact manifold M. The focus will be on how a suitable convolution semi-group of probability measures on the diffeomorphism group of M leads first to Gross-Sobolev spaces of functions on the paths on the diffeomorphism group with an associated Malliavin calculus, and from then to such a calculus on C_xM, with the rudiments of an L_2 deRham-Hodge-Kodaira theory for forms on that infinite dimensional manifold. The standard Malliavin calculus for Brownian motion measure when M is Riemannian is a well studied, but not yet well understood, important special case. The role of the (partial) connection induced over M by the original convolution semi-group, and its associated connections on C_xM will be emphasised. Particular attention will be given to the special case arising when M is a Riemannian symmetric space. The talk draws on ideas of many different people, but the specific approach taken derives from work of S. Fang & J. Franchi, and collaboration with S. Aida, with Xue-Mei Li, and with Xue-Mei Li & Yves LeJan.

The Global Geometry of Stochastic Loewner Evolutions

Roland Friedrich

Max-Planck-Institute für Mathematik

In this talk we shall give a comprehensive picture of the mathematics in which Stochastic Loewner Evolutions are embedded.

In particular we shall relate the underlying probability theory with Integrable Systems, the Sato-Segal-Wilson Grassmannian, the representation theory of the Virasoro algebra and Teichmueller theory.

The talk is based on current and past results.

Rough path condition for smooth paths

Keisuke Hara

Ritsumeikan University

Rough path theory is usually applied to non-smooth, actually very irregular functions like Brownian paths or much more singular ones. Though it is just because we have great interest in such important objects, in technical sense it is also because we work on a compact interval and because differential paths trivially satisfy rough path condition on it. However, if we consider a path on the whole real line, it no longer has a trivial bound for the condition. For example, let us consider a one dimensional smooth function on the real line. Then, the total variation or the p-variations of the path can be infinite because the length of the path can be already infinite and the uniform estimate refers to a function on the whole real line. Therefore, we can ask what the natural conditions are for the bounded variations of smooth paths on the real line.

The main aim of my talk is to show a general version of the question above and to give an answer. We will suppose that a smooth path has an integrability of the path itself and the derivative, and we will estimate the p-variations of the iterated integrals in the framework of general rough path theory. In other words, we will see that there exists a non trivial "smooth rough path". The point is to control the behaviour of the global oscillation using uniform estimate on the control function.

We will also apply the rough path property of a smooth path to study the global behaviour, especially the limiting behaviour of the path itself or the solutions of a differential equations driven by the path as the time goes infinity. This scheme is almost automatic thanks to the generality of rough path theory. But we need to be careful because we apply the argument to a non-compact time interval.

We will basically work with infinitely differentiable paths, so we can skip almost all subtle points related to much singularity of paths in usual rough path theory. But it shows yet another application of rough path theory with a little different flavor.

Reference:
K.Hara and T. Lyons "Smooth rough paths and applications to Fourier analysis", Rev. Mat. Iberoamericana 23 (2007), no.3, pp.1125–1140.

A stochastic Taylor-like expansion in the rough path theory

Yuzuru Inahama

Tokyo Institute of Technology

We will prove a stochastic Taylor-like expansion for a differential equation in the framework of the rough path theory. As an application, we prove a Laplace approximation for the loop group-valued Brownian motion.

Since matingale integration theory does not work very well in a general Banace space, the rough path theory may be a useful tool to investigate the Banach space-valued processes.

A fixed-point property of discrete groups and an energy of equivariant maps

Hiroyasu Izeki

Mathematical Institute, Tohoku University

Let Γ be a finitely generated group. We say Γ has the fixed-point property for a metric space Y (or a class of metric spaces $\mathcal{Y}$) if every isometric action on Y (or each $Y \in \mathcal{Y}$) admits a global fixed point. (For example, the fixed-point property for Hilbert spaces is known to be equivalent to Kazhdan's property (T).)

Given a irreducible symmetric random walk on Γ, we can define an energy $E(f)$ of an equivariant map f from Γ into a space Y on which Γ acts isometrically. We give a certain sufficient condition for Γ to have the fixed-point property for a CAT(0) space Y (or a class of CAT(0) spaces $\mathcal{Y}$) in terms of the energy of equivariant maps. We also present some fixed-point theorems for finitely generated groups derived from this sufficient condition.

Random graphs and equivalence relations

Vadim Kaimanovich

Jacobs University Bremen

The theory of graphed equivalence relations provides a natural framework for considering random graphs. The talk is devoted to the links between the structural properties of the equivalence relations and the probabilistic properties of the associated Markov chains.

Functions of finite Dirichlet sum and compactifications of infinite graphs

Atsushi Kasue

Kanazawa University

In this talk, we consider a connected, infinite graph of bounded degree and discuss some geometric and potential theoretic properties of the graph.

Given a family of bounded functions on a graph, there exists a (up to canonical homeomorphism) compact Hausdorff space in which the graph is embedded as an open and dense subset such that every function of the family extends to a continuous function on the compact space and the extended functions separate the points of the boundary. For instance, by the family of bounded functions that are locally constant outside a compact subset of the graph, we have the end compactification of the graph. The compact Hausdorff space associated to the space of functions of finite Dirichlet sum of order p (>1) is called the p-Royden compactification of the graph, which forms a non-decreasing series of compact Hausdorff spaces indexed by the exponents over the graph. The set of regular points of p-Royden boundary is called the p-harmonic boundary and points of the boundary are separated by the traces of p-harmonic functions of finite Dirichlet sum of order p. This compactification is complicated in general; for instance, the set of an irregular point, if it exists, is not a G-delta set. It may be the case that the boundary consists of only one point for any exponent p.

We note that a qusi isometry between two graphs induces a homeomorphism between the p-Royden boundaries and also their p-harmonic boundaries. We also observe that any map of a graph to a metric space with compact boundary separated by the distance extends to a continuous map from the p-Royden boundary to the boundary of the target space if the Dirichlet sum of order p of the map is finite. This will be useful to investigate the p-Royden compactification of the graph and its relations to other ones.

A dynamical pattern formation, tropical geometry and informative entropy

Tsuyoshi Kato

Department of Mathematics, Faculty of Science, Kyoto University

In this talk, I will propose a mathematical formulation of pattern formation by use of dynamical systems, passing through tropical geometry. We show that informative entropy fits with analyzing its underlying structure.

Orbit equivalence rigidity for some groups acting on trees

Yoshikata Kida

Mathematical Institute, Tohoku University

I present a class of amalgamated free products which satisfy rigidity properties for their probability-measure preserving actions in terms of orbit equivalence. One consequence is that if an action of such an amalgamated free product is orbit equivalent to an arbitrary action of an arbitrary group, then the latter group acts on the Bass-Serre tree associated to the former group splitting and it admits a similar splitting.

Measurable Riemannian geometry on the Sierpinski gasket

Jun Kigami

Graduate School of Informatics, Kyoto University

We study the standard Dirichlet form and its energy measure, called the Kusuoka measure, on the Sierpinski gasket as a prototype of "measurable Riemannian geometry". The shortest path metric on the harmonic Sierpinski gasket is shown to be the geodesic distance associated with the "measurable Riemannian structure". The Kusuoka measure is shown to have the volume doubling property with respect to the Euclidean distance and also to the geodesic distance. Li-Yau type Gaussian off-diagonal heat kernel estimate is established for the heat kernel associated with the Kusuoka measure.

Ricci and flag curvatures in Finsler geometry

Chang-Wan Kim

Korea Institute for Advanced Study

In this talk, I discuss Finsler metrics with positive constant flag curvature. The natural invariant one defines in Finsler geometry is the invariant that governs the variation of these geodesics, i.e., the Jacobi fields. A Finsler manifold is said to have constant flag curvature if its Jacobi operator along any geodesic is conjugate to that along a geodesic in a Riemannian space form of constant sectional curvature. Among the more recent interesting results have been the determination of these structures and, via some recent results of Alvarez Paiva, the proof that the Riemannian round sphere is the only reversible Finsler metric of constant flag curvature on the sphere.

Fixed-point property of random groups

Takefumi Kondo

Kyoto University

We prove that various random groups have fixed-point property for a certain class of CAT(0) spaces. This class contain all symmetric spaces of non-compact type, Hilbert spaces and some Euclidean buildings. As an application, we prove that in a space of marked groups, there are many groups which is non-linear.

Characterization of maximal Markovian couplings for diffusion processes

Kazumasa Kuwada

Ochanomizu University

Necessary conditions for the existence of a maximal Markovian coupling of diffusion processes are studied. A sufficient condition described as a global symmetry of the processes is revealed to be necessary for the Brownian motion on a Riemannian homogeneous space. As an application, we find new examples of a non-maximal Kendall-Cranston coupling. We also find a Markov chain admitting no maximal Markovian coupling for specified starting points.

On discrete Harmonic maps into CAT(k)-spaces via Markov chains

Kazuhiro Kuwae

Graduate School of Science and Technology, Kumamoto University

I will talk about the existence and construction of harmonic maps into CAT(k)-spaces having diameter strictly less than $\pi/2\sqrt{k}$ and convex geometry. The notion of harmonic maps is based on discrete time conservative Markov processes and can be defined in terms of barycenter of transition kernel over such spaces. The theory of discrete time martingales taking values into such spaces plays an important role. I also report that the Jensen's inequality for probability measures over such spaces holds, which is recently proved by the speaker.

Optimal transport and Perelman's reduced volume

John Lott

University of Michigan

We show that a certain entropy-like function is convex, under an optimal transportation problem that is adapted to Ricci flow. We use this to reprove the monotonicity of Perelman's reduced volume.

Statistical manifolds and affine differential geometry

Hiroshi Matsuzoe

Nagoya Institute of Technology

A statistical manifold is a Riemannian manifolds with a totally symmetric tensor field of order 3. From the given Riemannian metric and the symmetric tensor field, a pair of torsion-free affine connections can be defined on a statistical manifold. Since such geometric structures are naturally arisen in geometry of statistical models, a statistical manifold is one of fundamental geometric objects in information geometry. On the other hand, statistical manifolds are induced from affine immersions. Therefore, geometry of statistical manifolds is an intersection between information geometry and affine differential geometry.

In this talk, we discuss geometry of statistical manifolds from the viewpoint of affine differential geometry. Dually flat structures and canonical divergence on flat statistical manifolds are useful objects in information geometry. Such geometric structures are induced from affine immersions and affine support functions. In addition, information geometry suggests a duality of conformal or projective structures. We elucidate a duality of conformal structures in terms of affine immersions.

We also discuss generalizations of statistical manifolds. It is known that affine connections on quantum state spaces in quantum information theory have non-zero torsions. Hence we have to generalize (classical) statistical structures if we discuss quantum information geometry. In this talk, we will mention geometry of statistical manifolds admitting torsion.

Curvature, continuity and uniqueness of optimal transportation maps

Robert J. McCann

Department of Mathematics, University of Toronto

Despite years of study, surprisingly little is understood about the optimal transportation of a mass distribution from one manifold to another, where optimality is measured against a cost function on the product space.

I shall present a uniqueness criterion subsuming all previous criteria, yet which is among the very first to apply to smooth costs on compact manifolds, and only then when the topology is simple. This new result is based on a characterization of the the support of an extremal double stochastic measures.

As time permits, I shall also review my surprising discovery with Young-Heon Kim (University of Toronto) that the regularity theory of Ma, Trudinger, Wang and Loeper for optimal maps is based on a hidden pseudo-Riemannian structure, which leads to a simple direct proof of a key result in the theory, and opens several new research directions.

Optimal transport and Ricci curvature in Finsler geometry

Shin-ichi Ohta

Department of Mathematics, Kyoto University

We introduce a new notion of Ricci curvature of a Finsler manifold (equipped with an arbitrary measure) inspired by the theory of weighted Riemannian manifolds.

Then we discuss optimal transport in Finsler manifolds, and extend many results from Riemannian to Finsler manifolds. Among them, we generalize the Brascamp-Lieb inequality, the Brunn-Minkowski inequality and the equivalence between Lott, Sturm and Villani's curvature-dimension condition and the lower Ricci curvature bound. There are applications to functional inequalities as well as the concentration of measure phenomenon.

Statistical mechanics of 1-particle ideal gas and deformation of Alexandrov spaces

Yukio Otsu

Department of Mathematics, Kyushu University

In this talk, first on a fixed compact Alexandrov space we consider M-tuple of random points, which we call random net. Then we associate a discrete Laplacian with each net and take ramdon average, which converge to the continun Laplacian as $M \to \infty$. We consider every random nets of every compact Alexandrov spaces and associate the discrete Laplacians with each net, which are acting on the same Hilbert spaces. In this way we can interpret deformation of Alexandrov spaces as perturbation of Laplacians.

Next we consider the equilibrium statistical mechanics of 1-particle ideal gas associated with each discrete Laplacian, because we can construct density matrix of these system by interpreting the Laplacian as the Hamiltonian of 1-particle ideal gas. Then we consider free energy of these system, and prove its concavity under the isothermal condition. As a result we can construct a family of non-negative quadratic forms on operator spaces, which induces a family of Riemannian metrics on the spaces of random nets. Finally we reconstruct deformation of Alexandrov spaces from this point of view.

Urysohn's universal, or random, metric space, its group of isometries, and other related structures

Vladimir Pestov

University of Ottawa

The universal metric space $\mathbf{U}$ was defined by Pavel Urysohn in 1924 as a unique complete separable metric space that is ultrahomogeneous (each isometry between two finite subsets extends to a global self-isometry of $\mathbf{U}$) and contains a copy of every separable metric space. This is a refined version of Rado's random graph R, which was introduced much later. In particular, Vershik had shown that $\mathbf{U}$ can be interpreted as both a random and a generic separable metric space. Some properties of $\mathbf{U}$ are astonishing to the point of being hard to believe, such as the theorem of Holmes: if a normed space E contains an isometric copy of $\mathbf{U}$ which spans E and includes zero, this is enough to determine the norm of every element of E in a unique way.

Analogues of some known properties of R are either unknown for $\mathbf{U}$ (e.g. the existence of a model), or proved to be very hard. For instance, oscillation stability, an analogue of indivisibility of R, has only been very recently established by Nguyen Van Thé and Sauer, and their remarkable result shows that the unit sphere in the Urysohn space behaves very differently from the unit sphere in the Hilbert space, which has distortion according to the well-known result by Odell and Schlumprecht.

Yet further properties of $\mathbf{U}$ have no analogues for R: for instance, V.V. Uspenskij's theorem that the isometry group $Iso\mathbf{U}$ is a universal Polish group.

We will survey the above results and other recent developments.

Non symmetric diffusions on a Riemannian manifold

Ichiro Shigekawa

Department of Mathematics, Graduate School of Science, Kyoto University

We discuss non-symmetric diffusions on a Riemannian manifold. We are interested in the diffusions whose generator is of the form sum of the Laplace-Beltrami operator and a vector field. We give a sufficient condition for that the associated semigroup is a C_0 semigroup in L^2. We also determine the domain of the genrator. We also give a sufficient condition for that the associated semigroup is C_0 semigroup in L^p with $p < 2$.

Geometric analysis on Alexandrov spaces

Takashi Shioya

Mathematical Institute, Tohoku University

An Alexandrov space is a metric space of curvature bounded below. A typical example of Alexandrov spaces is the Gromov-Hausdorff limit of Riemannian manifolds of sectional curvature bounded from below. Sometimes, Alexandrov spaces are useful to study the Gromov-Hausdorff convergence/collapsing phenomena under a lower bound of sectional curvature. In this talk, I survey the study of geometric analysis on Alexandrov space, including the existence of the heat kernel, the Laplacian comparsion theorem, and a splitting theorem under the infinitesimal version of volume comparison condition.

Bernstein measures on convex polytopes

Tatsuya Tate

Graduate School of Mathematics, Nagoya University

We define the notion of Bernstein measures and Bernstein approximations over general convex polytopes. This generalizes well-known Bernstein polynomials which are used to prove the Weierstrass approximation theorem on one dimensional intervals.

We discuss some properties of Bernstein measures and approximations, and prove an asymptotic expansion of the Bernstein approximations for smooth functions which is a generalization of the asymptotic expansion of the Bernstein polynomials on the standard m-simplex obtained by Abel-Ivan and Hörmander.

These are different from the Bergman-Bernstein approximations over Delzant polytopes recently introduced by Zelditch. We discuss relations between Bernstein approximations defined in this paper and Zelditch's Bergman-Bernstein approximations.

Li-Yau type inequalities and a priori estimates for heat equations by Stochastic Analysis

Anton Thalmaier

Université du Luxembourg

In this talk we focus on localized versions of gradient estimates and Harnack type inequalities for solutions of heat equations. We show that methods from Stochastic Analysis may serve as a unifying tool. The talk is based on joined work with Marc Arnaudon, Bruce Driver and Feng-Yu Wang.

Random matrices, probability, and geometry

Bálint Virág

University of Toronto

Random hermitian matrices have eigenvalues that repel each other on short distances. I will discuss how this repulsion can be understood through Brownian motion in the hyperbolic plane.

Entropic Measure and Wasserstein Diffusion

Max von Renesse

Technische Universität Berlin

We present the construction of a natural diffusion process on the space of probability measures which can be interpreted as a Riemannian Brownian motion plus drift on the Wasserstein space.

Crucial to our construction is the existence of a invariant measure on the space of probability measures which has a Gibbs structure with the relative Entropy funtional as Hamiltonian.

What we can estimate about a singularity from random samples

Sumio Watanabe

Tokyo Institute of Technology

In information science, sometimes we need to estimate a singularity from random samples. In such a case, neither the maximum likelihood estimator has the asymptotic normality nor Bayes a posterior distribution converges to the normal distribution. Therefore we have an elemental question, how much information about a singularity we can extract from a finite set of random samples. In this presentation, we introduce two birational invariants about a singularity, a real log canonical threshold and a singular fluctuation. We prove that two universal equations hold about two invariants for any true distribution, any statistical model, any a priori distribution, and any singularities. By using these two equations, we show that two invariants of a singularity can be estimated from random samples without any knowledge of information source.

Automata groups

Andrzej Zuk

Université Paris 7

We will present recent developments in the theory of groups generated by finite automata.

Contributed Talks and Posters

Time-dependent Backward Stochastic Evolution Equations

AbdulRahman S. Al-Hussein

Department of Mathematics, Qassim University

Let us consider the following infinite dimensional backward stochastic evolution equation:

$$\begin{cases} -dY(t) = (A(t)Y(t) + f(t, Y(t), Z(t)))dt - Z(t)dW(t), \\ Y(T) = \xi, \end{cases}$$

where W is a cylindrical Wiener process on a separable Hilbert space and $A(t), t \geq 0$, are unbounded operators that generate a strong evolution operator $U(t, r), 0 \leq r \leq t \leq T$. We prove under non-Lipschitz conditions that such an equation admits a unique evolution solution. Some examples and regularity properties of this solution are given as well.

On Topological Obstructions of Compact Riemannian and Combinatorial Positively Ricci Curved Manifolds

Wen-Haw Chen

Department of Mathematics, Tunghai University

We will first give new information on obstructions to the fundamental groups of compact positively Ricci curved Riemannian manifolds, and it extends the classical Myers' theorem. Moreover, Rubin Forman introduced a purely combinatorial notion of Ricci curvature for cell complexes depending only on the relationships between the cell and its neighbors. Interestingly, Myers' theorem has its analogue in this combinatorial setting. We will also give obstructions to the topology of compact combinatorial manifolds with positively combinatorial Ricci curvature.

Brownian survival among perturbed lattice traps

Ryoki Fukushima

Division of Mathematics of Kyoto University

We consider Brownian motion among killing traps attached around randomly perturbed lattice. We derive the annealed asymptotics for the survival probability, which depends on the tail of the perturbation variables. From our results, it follows that the asymptotic order approaches to the periodic regime in the weak disorder limit and to the Poissonian regime in the strong disorder limit.

Concentration of 1-Lipschitz maps and group action

Kei Funano

Mathematical Institute, Tohoku University, Sendai

I will talk about Lévy-Milman concentration theory of 1-Lipschitz maps and its application to Lévy group action.

Weyl type spectral asymptotics for the Laplacian on Sierpinski carpets

Naotaka Kajino

Kyoto University

In this talk, I will talk about a detailed asymptotic behavior of the eigenvalues of the Laplacian on Sierpinski carpets.

There are many results for construction of Laplacians on self-similar sets, and it is known that there exist good strong local regular Dirichlet forms on a class of finitely ramified self-similar sets (nested fractals, such as the Sierpinski gasket) and also on a class of Sierpinski carpets.

For such a Dirichlet form on a self-similar set, the corresponding non-negative self-adjoint operator ('Laplacian') is shown to have compact resolvent, so its spectrum is written uniquely in the form of a non-decreasing sequence tending to infinity. The *eigenvalue counting function* $N(x)$ of this Laplacian is defined as the number of eigenvalues less than or equal to x. We would like to know the asymptotic behavior of $N(x)$ as x tends to infinity.

For finitely ramified self-similar sets, such kinds of results were known already in 90's. Recently for the Laplacian on Sierpinski carpets, B. M. Hambly (Asymptotics for functions associated with heat flow on the Sierpinski carpet, preprint) has proved a similar result for the trace (say $T(t)$) of the corresponding heat semigroup instead of $N(x)$, using arguments on the diffusion process and the sub-Gaussian estimate of the transition density. All these results are concerning the principal term of the asymptotic behavior of $N(x)$ (as x tends to infinity) or $T(t)$ (as t tends to 0).

The main result of this talk concerns the asymptotic behavior (as t tends to 0) of

$$T(t)\text{-}\textit{(the principal term)},$$

namely: For the case of the Laplacian on nested fractals or Sierpinski carpets, $T(t)$-*(the principal term)* admits an asymptotic behavior similar to the principal term (and the same is partly true for higher order terms).

Riesz transforms on a path space with Gibbs measures

Hiroshi Kawabi

Department of Mathematics, Faculty of Science, Okayama University

In this talk, we discuss L^p-boundedness of the Riesz transforms on an infinite volume path space $C(\mathbf{R}, \mathbf{R^d})$ with Gibbs measures. Our approach is based on the Littlewood-Paley-Stein inequalities, and we will point out the gradient estimate of the corresponding diffusion semigroup plays a crucial role for these inequalities.

The Runege theorem for instantons

Shinichiroh Matsuo

University of Tokyo

A classical theorem of Runge asserts that a meromorphic function defined on a domain in the complex plain can be approximated, over compact subsets, by rational functions, i.e. by meromorphic functions on the entire Riemann sphere. In this talk we shall present an analogous result in which meromorphic functions are replaced by instantons over orieted 4-manifolds. This generalizes Donaldson's "Runge theorem for instantons over the four-sphere".

Synthetic Ricci curvature bounds in the Heisenberg group

Juillet Nicolas

University of Bonn

The Measure Contraction Property (MCP) and the Curvature-Dimension condition (CD) are two properties for metric measure spaces that replace lower bounds for the Ricci curvature (that is just defined on Riemannian manifolds). We will analyze these two approaches for the subRiemannian Heisenberg group.

On Wasserstein geometry of the space of Gaussian measures

Asuka Takatsu

Mathematical Institute, Tohoku University

The goal of this poster is to investigate of geometry of the space of Gaussian measures on Euclidean space (we call the space of Gaussian measures "Gaussians" for short). In particular, we introduce a formula for the sectional curvatures of Gaussians on the Euclidean space using the Riemannian metric whose Riemannian distance coincides with the Wasserstein distance. This formula implies that the sectional curvature of Gaussians is represented in terms of the eigenvalues of the covariance matrix only.

A Gaussian measure is an absolutely continuous measure with respect to the Lebesgue measure. To be precise, its density is parameterized by mean m and covariance matrix V, where m is a vector of $\mathbf{R}^d$ and V is a symmetric, positive definite, matrix of size d. If we consider a subspace of Gaussians whose means are 0 and covariance matrices are $2tE$, then the family is the heat kernel.

Given a complete Riemannian manifold M, we shall denote by $P_2(M)$ the set consisting of measures having finite second moment on M. The (quadratic) Wasserstein distance W_2 is a distance on $P_2(M)$, which is defined by the Monge-Kantrovich optimal transport problem. We call the infinite dimensional metric space $(P_2(M), W_2)$ Wasserstein space. The Wasserstein space also inherits several properties of the base manifold M. For example, the lower Ricci curvature bound of M is linked to the stability of the heat equation. Thus the heat kernel is one of the most important subspace of the Wasserstein space.

Since the heat kernel on Euclidean space is parameterized by only by the parameter t, the subspace is flat by definition. Instead we analyze Gaussians on $\mathbf{R}^d$ which is considered as generalized heat kernels. Then we can regard Gaussians as a $d(d+3)/2$-dimensional manifold. Moreover, Gaussians is a geodesically convex subset of the Wasserstein space using the results from the Monge-Kantrovich transport theory; though an explicit expression of geodesics interpolating two measures is not unknown generally, it is known between Gaussians. The Wasserstein space over $\mathbf{R}^d$ is regarded as infinite dimensional manifold. By restricting to the space of Gaussian measures inside the space, we manage to provide detailed descriptions of the Wasserstein geometry from a Riemannian geometric viewpoint.

Concentration of measure via approximated Brunn-Minkowski inequalities

Masayoshi Watanabe

Mathematical Institute, Tohoku University

The uniform measure on the standard sphere concentrates around the equator as the dimension gets large. This is a typical example of the (Gaussian) concentration-of-measure phenomenon. It is known that the positivity of Ricci curvature implies the Gaussian concentration of measure.

In this talk we show that a weaker version of the Brunn-Minkowski inequality with volume distortion coefficient implies a Gaussian concentration-of-measure phenomenon. The Brunn-Minkowski inequality with volume distortion coefficient holds in metric measure spaces with positive Ricci curvature in the sense of Lott-Sturm-Villani. Our inequality makes sense in discrete spaces.

Excursions away from a regular point for one-dimensional symmetric Lévy processes without Gaussian part

Kouji Yano

Graduate School of Science, Kobe University

A sample path of a strong Markov process for which the origin is a regular point is divided into small pieces called excursions away from the origin. The law of excursions are characterized by a certain sigma-finite measure on the canonical path space, which is called the excursion measure. Several descriptions of the excursion measure for one-dimensional Brownian motion are well-known. Among others, Imhof's relation says that the excursion measure is equivalent to the three-dimensional Bessel process whose Radon–Nikodym density is given by the harmonic function of the killed Brownian motion. Our purpose is to study the excursion measure for one-dimensional symmetric Levy processes. We establish some of description formulae parallel to the Brownian case. Under some assumptions which imply no Gaussian part, we prove that the excursion process enters oscillatingly; This situation is completely different from that in the Brownian case where the excursion process cannot jump across the origin.

Perelman's reduced volume and Gap theorem for the Ricci flow

Takumi Yokota

University of Tsukuba

In his seminal paper, G. Perelman introduced the reduced volume of the Ricci flow. Its monotonicity can be seen as a Ricci flow version of Bishop-Gromov inequality. In this talk, I will show that an ancient solution to the Ricci flow whose asymptotic limit is sufficienly close to the maximum is isometric to the Eucledian space. This is a generalization of M. Anderson's result for Ricci flat manifolds.

Projections in the reproducing kernel Hilbert spaces and the conditional probabilities of determinantal point processes in discrete spaces

Hyun J. Yoo

Department of Applied Mathematics, Hankyong National University

We discuss the Gibbsianness of determinantal point processes defined on discrete spaces. We first introduce the known results on how the conditional probability densities of determinantal point processes, which is the crucial concept for the Gibbsianness of the measure, are related to the projections of vectors in reproducing kernel Hilbert spaces. Then we discuss some partial results for the cases where the defining kernel operators are projection operators.
