

Higher Dimensional Birational Geometry

**ADVANCED STUDIES
IN PURE MATHEMATICS 35**

Chief Editor: Eiichi Bannai (Kyushu University)

**Higher Dimensional Birational
Geometry**

Edited by

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Mathematical Society of Japan

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Preface

The present volume consists of four research articles contributed by participants of the international conference *Higher Dimensional Algebraic Varieties*, 1997, at the Research Institute of Mathematical Sciences (RIMS), Kyoto University during June 2–6 and 9–13, 1997.

The conference, held as part of the special year activities of RIMS during the academic year 1996/97 and cosponsored by the Mathematical Society of Japan, represented the high level of the birational geometry at the time. The invited speakers and their talks were:

- Y. Miyaoka (U. Tokyo), On a numerical characterization of \mathbf{P}^n .
- N. Nakayama (RIMS), Projective manifolds whose universal covering spaces are \mathbf{C}^n .
- A. Corti (U. Cambridge), Birationally rigid \mathbf{Q} -Fano hypersurfaces I (Excluding maximal singularities).
- M. Reid (Warwick U.), Birationally rigid \mathbf{Q} -Fano hypersurfaces II (Constructing birational involutions and untwisting).
- J. Kollár (Princeton U.), Polynomials with integral coefficients.
- F. Bogomolov (Courant Inst.), Weak Hironaka theorem.
- Y. Kawamata (U. of Tokyo), Unobstructed deformations and index 1 covers.
- S. Keel (1) (U. Texas, Austin), Eventual freeness in char p , with applications to M_g and to 3-fold MMP.
- L. Ein (U. Illinois at Chicago), Singularities of theta divisors and birational geometry of irregular varieties.
- S. Helmke (RIMS), On global generation of adjoint linear systems.
- K. Matsuki (Purdue U.), Surface log terminal singularities in positive characteristic.
- S. Mukai (RIMS), Degeneration of $\mathbf{P}^n \times \mathbf{P}^n$ with application to del Pezzo fibration.
- V. Alexeev (U. Georgia), Log canonical singularities and complete moduli.
- G. Sankaran (U. Bath), Abelian surfaces in toric 4-folds.
- S. Keel (2) (U. Texas, Austin), Rational curves on open surfaces I.
- J. McKernan (U. C. Santa Barbara), Rational curves on open surfaces II.
- S. Mori (RIMS), On Reid's conjecture on general elephants.
- M. Hanamura (Kyushu U.), Mixed motivic sheaves

In the second week, more informal talks were given:

- A. Corti, Introduction to the Sarkisov program.
- J. Kollar, Real algebraic threefolds.
- S. Keel and J. McKernan, Rational curves on open surfaces.
- V. V. Shokurov (Johns Hopkins U.), Log flips.

We believe that it would have been a great volume if we could have collected the lectures delivered at the conference. However, some of them had already been submitted elsewhere and some were a little too technical to be included in the ASPM series, which is supposed to be of expository nature. By these twofold reasons, the editors decided that this volume should not be the ordinary proceedings of the conference but a selection of independent full expositions on different topics of fundamental importance in algebraic geometry: moduli spaces of abelian surfaces, rational curves on algebraic varieties, 3-dimensional flips and the theory of elliptic fibrations.

The authors made their best to update the articles to fill the time lag between the conference and the publication, and the editors hope that this volume will provide accessible and systematic treatments of several aspects of recent exciting developments in algebraic geometry.

The editors express their gratitude to the contributors who have patiently awaited the belated publication, and apologise to the Editorial Board of the ASPM for the delay of the preparation and the unusual style of this volume.

Shigefumi Mori
Yoichi Miyaoka

*All papers in this volume have been refereed and are in final form.
No version of any of them will be submitted for publication elsewhere.*

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