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受賞者のことば：

The study of the maximum of the first eigenvalue for Riemannian metrics on a compact surface of given topology is a natural problem with several applications in different areas of Geometry and PDEs. For genus zero Hersch (1970) showed that the spherical metric is the only maximizing one and for general topology Yang and Yau give an explicit bound of the functional in terms of the genus of the surface.

Marcel Berger conjectured that for genus  $g=1$  the maximum occurs for the flat metric on the equilateral torus and, in a remarkable breakthrough, this was proved by Nadirashvili.

The problem for  $g=2$  has been considered by several authors but only recently Nayatani and Shoda succeeded in proving that for these metrics the maximum of the first normalised eigenvalue of the Laplacian is attained by the spherical Bolza surface and for the nearby ones.

In the general case, Petrides shows that from every extremal metric one obtains a minimal surface in the  $n$ -dimensional sphere by the first eigenfunctions. This brings these two branches of geometric analysis closer together.

Our contribution concerns the case of genus  $g=3$ . From Yang and Yau's result, for any metric on this surface, the first normalised eigenvalue is less than or equal to  $24\pi$  and we improve the bound till the value of  $16(4 - \sqrt{7})\pi \approx 21.668\pi$ . It is remarkable to note that for the most symmetric hyperbolic metric, the Klein's quartic, a mixture of theoretical and numerical estimates give  $\approx 21.414\pi$ .

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